Spillovers Across U.S. Financial Markets[1]

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Abstract

Movements in the prices of different assets are likely to directly influence one another. This paper develops a model that identifies the contemporaneous interactions between asset prices in U.S. financial markets by relying on the heteroskedasticity in their movements. In particular, we estimate a "structural-form GARCH" model that includes the short-term interest rate, the long-term interest rate, and the stock market. The results indicate that there are strong contemporaneous interactions between these variables. Accounting for this behavior is critical for interpreting daily changes in asset prices and for predicting the future paths of their variances and correlations. We demonstrate the importance of this consideration in a risk-management application.

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1 Introduction

Movements in the price of one asset are likely to be importantly affected by the contemporaneous movements of other assets. This behavior arises in part because asset prices are driven by underlying factors, such as macroeconomic developments, monetary policy expectations, or risk preferences, that likely affect one another. In addition, the movement in one asset price may itself have macroeconomic implications that affect the value of other asset prices simultaneously. For example, changes in equity prices have an impact on aggregate demand through the wealth channel, and thus those changes likely influence the expected path of monetary policy embedded in market interest rates. Conversely, changes in market interest rates directly influence stock prices by affecting the rate at which future dividends are discounted.

Because asset prices are likely to be so intertwined, analyzing any single market in isolation ignores important information about its behavior. Changes in the price of an asset will be driven not only by its own innovations, but also by its reaction to movements in the prices of other assets. Thus, to be able to effectively interpret the high frequency dynamics of financial markets, one has to estimate a model that measures the contemporaneous interactions between financial variables. Unfortunately, it is difficult to estimate these contemporaneous relationships, precisely *because* these variables endogenously respond to one another and to other variables in the economy.

This paper presents an empirical model that identifies the contemporaneous interactions between asset prices in U.S. financial markets by relying on their conditional heteroskedasticity. In particular, we estimate a "structural-form GARCH" model that includes the short-term interest rate, the long-term interest rate, and stock market returns. In this model, shifts in the conditional variances of the shocks to the financial variables have implications for the covariances between the variables that depend on their responsiveness to one another. Thus, the model is able to recover the contemporaneous interactions between the variables essentially by placing cross-restrictions on the evolution of the second moments of the asset prices over time.

The results indicate that there are strong contemporaneous interactions between stock prices, short-term interest rates, and long-term interest rates. Increases in stock prices tend to push up both short-term and long-term interest rates, likely reflecting the influence of equity prices on aggregate demand. Conversely, equity prices react negatively to increases in each of the interest rates, since they affect the rate at which future dividends are discounted. Regarding the yield curve, increases in the short-term interest rate tend to result in smaller increases in long-term rates, thus causing the yield curve to flatten. The short-term rate in turn responds positively to increases in long-term rates, perhaps because those movements signal higher future inflation. In addition to these contemporaneous interactions, the model finds rich dynamics in the volatilities of the structural shocks, including some spillover of volatility from one market to another.

In contrast to the model that we develop, almost all of the GARCH models from the existing literature are "reduced-form GARCH" models, in that they do not attempt to measure the contemporaneous reactions of asset prices to one another. As a result, it is more difficult to interpret the innovations to the financial variables in those models. The structural-form GARCH model allows one to determine the source of the current movement in a given asset price—that is, whether it was driven by a shock to the asset itself, or by the endogenous response to a shock to another asset price innovation. Determining this source is of great importance for predicting the future paths of the variances of the financial variables and the correlations between them.

The importance of this consideration can be illustrated in a simple example in which there is an unexpected increase in the short-term interest rate. This increase could be driven by a shock to the interest rate itself, or by the endogenous response of the interest rate to a positive shock to equity prices. If the movement were initiated by an equity shock, then financial markets are likely to experience sizable equity shocks going forward (because the volatility of equity shocks is persistent). This would generate a positive covariance between future movements in stock prices and the interest rate, since equity shocks pull the interest rate in the same direction. By contrast, if the shock originated from the interest rate, then one would expect a negative correlation going forward, as financial markets would be more likely to experience additional interest-rate shocks, which push equities in the opposite direction. The structural-form GARCH model separates these possibilities by looking at the contemporaneous movements across various asset prices.

As the example makes clear, an important implication of the model is that correlations between asset prices can shift substantially as the relative volatilities of the underlying shocks change.[1] The results presented below indicate that variation in these correlations is an important aspect of the behavior of the financial variables considered. Indeed, one of the findings is that the correlation between equity prices and Treasury yields has become positive in recent years due to the increase in the volatility of equity market shocks. This positive correlation has been widely discussed by market participants.[2]

Because it captures such behavior, the model presented has compelling implications for asset pricing and risk management. Failing to accounting for the influence of other asset prices could lead to significant mismeasurement of the risk of a given portfolio. Moreover, because it recovers the shocks that are driving the system, the model allows one to better predict the peristence of the volatility of asset prices, with obvious implications for determining the value of options and other financial instruments.

The paper begins by specifying the structure of the model in section 2 and comparing it to the types of GARCH models that have been estimated in the literature to date. Section 3 presents the results, with extensive discussion of the contemporaneous interactions between the financial variables and possible interpretations. That section also demonstrates the importance of recovering the structural-form shocks for predicting the correlations between financial variables. Section 4 presents an application of the model to a risk management problem, demonstrating that the predictions of the model have sizable practical implications, and section 5 concludes.

2 Identifying Interactions between Asset Prices

We are interested in estimating the relationships between three financial variables: the three-month Treasury bill rate (i_t), the ten-year Treasury yield (measured by the slope of

[1]The extent to which reduced-form multivariate GARCH models can capture this behavior varies across the specifications used. These models, and the advantages that the structural-form GARCH model has over them, are discussed in more detail below.

[2]For example, a research report by Goldman Sachs(2002) notes that, "Fixed income market participants are paying more and more attention to developments in equity markets. Subjectively, it often seems that every time stocks go down, the bond market rallies – and vice versa."

the Treasury yield curve from three months to ten years, y_t), and the return on the S&P 500 stock market index (s_t). Our focus is on measuring the high frequency dynamics between these variables, and so the data used are at a daily frequency. Clearly, the methodology developed can be easy extended to include additional variables—such as exchange rates, commodity price indexes, and foreign asset prices.

2.1 The Structural-form GARCH Model

We assume that the dynamics of the three asset prices are described by the following structural-form model:

$$A \underset{3\times T}{x_t} = \psi + \Phi(L)x_t + \phi(L)z_t + \eta_t, \tag{1}$$

where $x_t \equiv \{i_t, s_t, y_t\}'$. The matrix A captures the contemporaneous relationship among the three financial variables, which is of primary interest to this paper. The matrix is normalized to have the following form:

$$\underset{3\times 3}{A} \equiv \begin{pmatrix} 1 & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & 1 & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & 1 \end{pmatrix}, \tag{2}$$

where the off-diagonal elements are coefficients measuring the contemporaneous interactions between the variables included in x_t. As for the rest of the system (1), ψ is a vector of constants, and $\Phi(L)$ is a lag function that controls for the lags of the endogenous variables. The specification also allows other exogenous variables, denoted z_t, to affect these financial variables, which could include macroeconomic variables or commodity prices, for example.

Equation (1) essentially is a latent-factor model of asset price movements. The vector $\eta_t \equiv \{\eta_{i,t}, \eta_{s,t}, \eta_{y,t}\}'$ represents the "structural shocks" to the system—or the innovations to the latent factors that drive asset price movements, which are discussed in more detail below. We assume the conditional expectations of the structural shocks are zero. Moreover, because they represent innovations to fundamental factors rather than reduced-form innovations, we assume that their conditional cross-moments are zero at all leads and lags:

$$E_t(\eta_i) = 0,\ E_t(\eta_s) \quad = \quad 0,\ E_t(\eta_y) = 0, \tag{3}$$

$$E_t(\eta_i \eta_s) = 0,\ E_t(\eta_i \eta_\pi) \quad = \quad 0,\ E_t(\eta_y \eta_s) = 0, \tag{4}$$

for all t. In addition, we assume that the variances of the structural shocks exhibit conditional heteroskedasticity in the same manner that GARCH models assume for the reduced-form shocks:

$$\eta_i \sim \sqrt{h_i} \cdot \zeta_i \tag{5}$$

$$\eta_s \sim \sqrt{h_s} \cdot \zeta_s \tag{6}$$

$$\eta_y \sim \sqrt{h_y} \cdot \zeta_y \tag{7}$$

$$h_t \quad = \quad \psi_h + \underset{3\times 3}{\Gamma} h_{t-1} + \underset{3\times 3}{\Lambda} \eta_{t-1}^2, \tag{8}$$

where ζ_i, ζ_s, and ζ_y, are three independent shocks that are normally distributed with mean zero and unitary variance. The conditional variances of the structural shocks are then given by the vector $h_t \equiv \{h_{i,t}, h_{s,t}, h_{y,t}\}'$. Following the prototypical GARCH set-up, these conditional variances are assumed to evolve based on their lagged values, the magnitudes of the most recent shocks, and a vector of three constants ψ_h.

The matrices Γ and Λ, which determine the dependence of the variances on their lagged values and on lagged shocks, are subject only to the restrictions that their elements have to be positive and that they have to imply finite second moments.

2.2 Identification

In general, it is difficult to estimate the responsiveness of asset prices to one another (the matrix A) due to the issue of endogeneity, given that each asset price likely responds to the other asset prices included in the specification. However, identification can be achieved when there is conditional heteroskedasticity in the data.

The intuition for the identification is straightforward: as the conditional variances of the structural shocks shift over time, the conditional covariances between the variables must shift in a manner that depends on their contemporaneous responsiveness to one another. This is shown for two asset prices in Figure 1, which assumes (as suggested in the introduction) that the short-term interest rate has a negative impact on equity prices, and that equity prices have a positive impact on the interest rate. In that case, periods during which shocks to the stock market are more volatile than shocks to the interest rate induce a positive correlation between the variables (top panel), while periods in which the interest rate shock is relatively more volatile induce a negative correlation (bottom panel).

This identification methodology is similar to that pursued in Rigobon and Sack (2003), which focused on measuring the response of the short-term interest rate to the stock market.[3] In that paper, we defined discrete regimes for the variances of stock market and interest rate shocks and relied on the shift in the covariance matrix of the shocks across regimes to identify the response coefficient. The current paper instead allows the variances of the shocks to evolve in a continuous manner—which, in essence, provides a continuum of regimes for identifying the contemporaneous responses. Moreover, under this approach we can simultaneously estimate the responses of all of the financial variables to one another.

To estimate the model, we begin by deriving the reduced form of specification (1), which is obtained by premultiplying the system by A^{-1}:

$$x_t = c + F(L)x_t + f(L)z_t + v_t, \tag{9}$$

where the reduced form residuals v_t are given by

$$v_t = \begin{pmatrix} v_{i,t} \\ v_{s,t} \\ v_{y,t} \end{pmatrix} = A^{-1} \begin{pmatrix} \eta_{i,t} \\ \eta_{s,t} \\ \eta_{y,t} \end{pmatrix}.$$

The coefficients in equation (9) can be estimated by OLS and are related to the structural-

[3]The intuition for this identification method was first introduced by Wright (1928). Recently, it has been explored by Sentana and Fiorentini (2001) and Rigobon (2002) in the case of conditional heteroskedasticity and by Rigobon (2003) in the case of unconditional heteroskedasticity.

form coefficients by

$$c = A^{-1}\psi, \quad F(L) = A^{-1}\Phi(L), \quad f(L) = A^{-1}\phi(L).$$

Thus, the structural coefficients can be recovered if A is identified.

If the structural shocks are described by the GARCH model assumed above, then the reduced form shocks v_t will exhibit GARCH behavior as well. More specifically, it can be shown that the second moments of the reduced-form residuals satisfy

$$\begin{aligned} v_t &\sim N(0, H_t) \\ H_t &= \begin{pmatrix} H_{ii,t} & H_{is,t} & H_{iy,t} \\ & H_{ss,t} & H_{sy,t} \\ & & H_{yy,t} \end{pmatrix}, \end{aligned}$$

and

$$\begin{pmatrix} H_{ii,t} \\ H_{is,t} \\ H_{ss,t} \\ H_{iy,t} \\ H_{sy,t} \\ H_{yy,t} \end{pmatrix} = \underset{6\times 3}{B_l} \cdot \psi_h + \underset{6\times 3}{B_l} \cdot \underset{3\times 3}{\Gamma} \cdot \underset{3\times 3}{(B^2)^{-1}} \begin{pmatrix} H_{ii,t-1} \\ H_{ss,t-1} \\ H_{yy,t-1} \end{pmatrix} + \underset{6\times 3}{B_l} \cdot \underset{3\times 3}{\Lambda} \cdot \underset{3\times 3}{(B^2)^{-1}} \begin{pmatrix} v_{i,t-1}^2 \\ v_{s,t-1}^2 \\ v_{y,t-1}^2 \end{pmatrix}, \tag{10}$$

where

$$B \equiv \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} = A^{-1}$$

and

$$B_l = \begin{pmatrix} b_{11}^2 & b_{12}^2 & b_{13}^2 \\ b_{11}b_{21} & b_{12}b_{22} & b_{13}b_{23} \\ b_{21}^2 & b_{22}^2 & b_{23}^2 \\ b_{11}b_{31} & b_{12}b_{32} & b_{13}b_{33} \\ b_{21}b_{31} & b_{22}b_{32} & b_{23}b_{33} \\ b_{31}^2 & b_{32}^2 & b_{33}^2 \end{pmatrix}.$$

These equations indicate that the structural-form GARCH specification (1) places restrictions on the evolution of the conditional variance-covariance matrix of the reduced-form innovations. Those restrictions arise from the fact that the variances and covariances of the reduced-form innovations must be consistent with the conditional heteroskedasticity of the structural-form innovations and the linear responsiveness of the financial variables to one another.

For comparison, note that the unrestricted reduced-form GARCH model analogous to equaton (10) is as follows:

$$\begin{pmatrix} H_{ii,t} \\ H_{is,t} \\ H_{ss,t} \\ H_{iy,t} \\ H_{sy,t} \\ H_{yy,t} \end{pmatrix} = \underset{6\times 1}{C_1} + \underset{6\times 3}{C_2} \begin{pmatrix} H_{ii,t-1} \\ H_{ss,t-1} \\ H_{yy,t-1} \end{pmatrix} + \underset{6\times 3}{C_3} \begin{pmatrix} v_{i,t-1}^2 \\ v_{s,t-1}^2 \\ v_{y,t-1}^2 \end{pmatrix}. \tag{11}$$

This specification potentially has a total of 42 parameters—6 constants in C_1, 18 elements in C_2, and 18 coefficients in C_3. Moreover, if equation (11) were completely unrestricted, the right-hand side would also include lagged covariance terms, which would add another 18 parameters to C_2–leaving a total of 60 parameters. By comparison, the structural-form model (10) contains only 27 parameters: the 3 constants ψ_h, 6 coefficients in the A matrix, and 9 coefficients in the Λ and Γ matrices.[4] Thus, the structure of our model imposes a number of restrictions on the reduced-form GARCH model. It is from these restrictions that we are able to recover the structural-form parameters.[5]

2.3 Comparison to Reduced-form GARCH

It may be useful at this point to compare our structural-form GARCH model to some of the multivariate GARCH models that have been estimated to date in the finance literature. It is important to note up-front that these models focus only on the reduced form (9), without recovering the matrix A. Thus, they do not measure the contemporaneous interactions between financial variables. Moreover, for this reason these models have to specify the conditional heteroskedasticity directly in terms of the reduced-form innovations v_t, rather than in terms of the structural-form shocks η_t.

The literature has proceeded by placing various restrictions on the form of the conditional heteroskedasticity in order to make the model tractable.[6] In many cases those restrictions are not derived, but are simply assumed in an ad-hoc manner. By contrast, the restrictions in our structural-form GARCH model arise from the assumption that the structural-form innovations are uncorrelated, which we believe is a reasonable constraint. Indeed, under the macroeconomic interpretations that we offer below, similar assumptions have been imposed in a vast amount of empirical macroeconomic research.

In comparing our model to the existing literature, we discuss three of the most common multivariate GARCH models—the Vech-GARCH, the constant correlation model, and the DCC-Garch model. The first of these, the Vech-GARCH model, assumes that the conditional moments depend only on the lag of the same conditional moment. This is equivalent to imposing a diagonal matrix for the lag conditional moments in the reduced-form vari-

[4]In addition, as will become evident below, a number of the off-diagonal elements of Λ and Γ are not significantly different from 0, which further reduces the number of parameters to be estimated. Indeed, a more parsimonious version of the model could be obtained by imposing that Λ and Γ are diagonal. However, in our application we find that a few of the off-diagonal elements of Λ are significant.

[5]Note that the model is identified only if there is conditional heteroskedasticity in the variables considered. Of course, there is an extensive literature indicating that this is the case.

[6]The completely unrestricted multivariate GARCH model has proven difficult to estimate because of the large number of parameters involved. Note that our model would become an unrestricted GARCH model if the structural shocks were allowed to have conditional covariances different from zero.

ance equation (the matrix C_2, only expanded to include coefficients on the lagged covariance terms as well). Notice that under the the structural-form GARCH model, this matrix is not diagonal (but it does collapse to only three columns).

The second approach, following Bollerslev (1990), is to assume that the conditional correlations between the included variables are constant over time. However, as will become evident below, one of the crucial implications of our model is that changes in the relative importance of various shocks (that is, in their conditional variances) generate considerable shifts in the conditional correlations between asset price movements. A model that assumes constant correlations obviously will not be able to capture this behavior.

Third, the Dynamic Conditional Correlation GARCH (DCC-GARCH) model of Engle (2002) is similar to our model in that it allows for intertemporal variation in the conditional correlation between asset prices. It does so by assuming that the correlation coefficient between two assets is driven by its own lag and the cross-product of the lagged reduced-form innovations. While this restriction makes the model tractable, the equation for the correlation coefficient must be assumed rather than derived.[7]

In summary, the structural-form GARCH model developed here has two advantages over the existing literature. First, it places some natural restrictions on the reduced-form GARCH model that might be more appealing than the ad-hoc restrictions of other models. And second, it provides estimates of the contemporaneous responses of financial variables to one another, which is an important step for interpreting developments in financial markets, as discussed below.

3 Results

The model is estimated using daily data over the sample November 1985 to March 2001. The three-month interest rate used is the constant-maturity Treasury bill rate reported on the Federal Reserve's H.15 data release. The yield curve slope is measured by the difference between that rate and the ten-year constant-maturity Treasury yield. The stock market

[7]Further adding to the tractability of the model, Engle (2002) argues that the estimation can proceed in two steps. In the first, one estimates univariate GARCH models for each of the individual variables. In the second, one uses those residuals and their conditional variances to estimate the dynamic behavior of the correlation coefficient.

return is measured by the daily change in the S&P 500 price index. All variables are measured in percentage points.

To estimate the structural-form GARCH, we first estimate the coefficients from the reduced-form VAR specification (9) by ordinary least squares, where the VAR includes five lags of the financial variables (x_t).[8] Note that these coefficient estimates are consisitent and provide us with the reduced-form innovations. Given the reduced-form innovations, we then obtain the parameters of the structural-form model by estimating equation (10) using maximum likelihood estimation (MLE). Under this approach, we obtain estimates of the 27 parameters contained in ψ_h, A, Λ, and Γ and the variance-covariance matrix of those parameters.

The contemporaneous interactions between the three financial variables are estimated to be as follows (with t-statistics in parentheses):

$$\begin{array}{rcl} i_t = & 0.0051 \cdot s_t & + & 0.3230 \cdot y_t \\ & (2.37) & & (8.59) \end{array} \tag{12}$$

$$\begin{array}{rcl} s_t = & -3.6422 \cdot i_t & - & 4.2990 \cdot y_t \\ & (-12.27) & & (-18.79) \end{array} \tag{13}$$

$$\begin{array}{rcl} y_t = & -0.7706 \cdot i_t & + & 0.0032 \cdot s_t. \\ & (-23.84) & & (2.15) \end{array} \tag{14}$$

The primary finding of this paper is that all of the contemporaneous response coefficients are significant, indicating that there are strong links across different financial instruments. Moreover, the parameters have the signs that one might have expected based on macroeconomic intuition. In the following paragraphs we offer some interpretations of these equations that may be useful for assessing the magnitudes of the estimated parameters.[9]

The equation for the short-term interest rate could be interpreted as a high-frequency monetary policy reaction function, as discussed in Rigobon and Sack (2003). This reaction function uses the three-month interest rate in order to capture daily changes in the near-

[8] The specification reported here does not include any exogenous variables (z_t). Including oil prices in the specification has virtually no impact on the results.

[9] Of course, the model is agnostic about the interpretation of the equations, as it simply requires that the system has the structure described in (1).

term outlook for monetary policy. That paper found that movements in equity prices generated responses in the short-term interest rate in the same direction. A similar result is found in (12). According to the estimates, a 1 percent increase in the S&P 500 index has a direct effect of 0.51 basis points on the short rate.[10] The results also suggest that the short rate responds considerably to the slope of the yield curve, reacting by about a third. One interpretation of this behavior is that an increase in the slope of the yield curve indicates expectations of higher future inflation, thus warranting a response in the short-term interest rate.[11] Under this interpretation, the shock $\varepsilon_{i,t}$ is analogous to what is often referred to as a "monetary policy shock," or a deviation from the typical response of the short rate.

In equation (14), the slope of the yield curve reacts negatively to the short-term interest rate, as movements in the short rate are apparently viewed as transitory, thus generating a smaller response of long rates. A 1 percentage point increase in the short rate results in a 77 basis point decline in the yield curve slope (or a 33 basis point rise in the long rate). The yield curve slope also reacts positively to increases in equity prices, perhaps because such increases boost aggregate demand and raise inflationary pressures. According to the equation, a 1 percent rise in equity prices has a direct impact of 0.32 basis points on the yield curve slope. The variable $\varepsilon_{y,t}$ represents an innovation to the yield curve slope, likely reflecting the impact of macroeconomic news on the outlook for inflation (or the inflation risk premium).

Lastly, equation (13) can be motivated as determining stock prices as the discounted sum of future dividends, as in various Gordon-type equations. Both the short rate and the yield curve slope enter this equation with negative coefficients, since they both increase the rates at which future dividends are discounted. According to the results, a 1 percentage point upward shift in the yield curve pushes down equity prices by 3.6 percent, while a 1 percentage point steepening has a negative impact of 4.3 percent.[12] The equity market

[10] This effect is somewhat smaller than the 2.1 basis point response found in Rigobon and Sack (2003). Some, but not all, of the difference is that there is an indirect effect that comes through long rates, as discussed below.

[11] The yield curve could also steepen in anticipation of higher real interest rates, for example due to expectations of more robust economic growth. However, the inflation expectations interpretation is also consistent with the strong estimated negative impact on equity prices.

[12] The contemporaneous impact of monetary policy shocks on equity prices is smaller than that found in Rigobon and Sack (2002) and Bernanke and Kuttner (2003). One reason might be that those papers focus

shock $\varepsilon_{s,t}$ could arise either from news that affects investors' expectations of future dividends or from a shift in investors' risk preferences.

Table 1: Estimates of Γ and Λ

Parameter	*Coefficient*	*Std. Error*
$\Gamma_{1,1}$	0.8076	0.0078
$\Gamma_{2,2}$	0.9535	0.0026
$\Gamma_{3,3}$	0.8298	0.0172
$\Lambda_{1,1}$	0.1420	0.0076
$\Lambda_{1,2}$	0.0002	0.0000
$\Lambda_{1,3}$	0.0246	0.0046
$\Lambda_{2,1}$	0.1770	0.0940
$\Lambda_{2,2}$	0.0393	0.0023
$\Lambda_{2,3}$	0	–
$\Lambda_{3,1}$	0.0321	0.0035
$\Lambda_{3,2}$	0	–
$\Lambda_{3,3}$	0.0328	0.0042

The remaining parameters of the model, which govern the conditional heteroskedasticity, are shown in Table 1. As can be seen, many of the parameters are found to be significant, which generates rich dynamics for the estimated variances of the structural shocks. We imposed that the elements of Λ and Γ had to positive, and the estimates of some of the off-diagonal coefficients hit this constraint.[13] The diagonal elements of Λ are found to be positive and significant, indicating that the conditional variance of each of the structural-form innovations tends to increase following a large innovation to that shock. Some of the off-diagonal elements of Λ are also significant, indicating that there are spillovers in volatility across the various shocks. In addition, the diagonal elements of the Γ matrix are

exclusively on FOMC dates, while the policy shocks used here include information from other dates that might have a smaller effect on stock prices.

[13]When we relaxed this restriction, it was impossible to find a solution in which the variances were assured to be positive for all realizations.

positive and significant, indicating that the variance of each structural shock also has an autoregressive structure.

The behavior implied by the model can be more clearly understood by looking at impulse response functions. The levels of the three financial variables have limited dynamics following each of structural shocks, as one might have expected given that asset price movements are largely unpredictable. Thus, we instead focus on the impulse response functions for the variances and covariances between the financial variables implied by the model. Figure 2 shows the responses of the conditional variances of the financial variables to a one-standard deviation shock to each of the three structural-form innovations. As can be seen, the conditional variance of each variable responds considerably to its own shock. But the variances also respond to other shocks as well. For example, the variance of the yield curve slope reacts most strongly to innovations to the short rate. In all cases the variances demonstrate some persistence, consistent with evidence from the existing GARCH literature.

Figure 3 shows similar impulse response functions for the conditional covariances between the financial variables. These covariances demonstrate a wide range of patterns in response to the various identified shocks, which result directly from the contemporaneous interactions in equations (12) to (14). For expositional purposes, we focus first on the covariance between stock prices and the short-term interest rate (the example discussed in the introduction). Shocks to the interest rate tend to make the covariance more negative over the subsequent several weeks. This effect arises because those shocks tend to be followed by additional interest rate shocks, which have a negative impact on equity prices. Shocks to the slope of the yield curve also induce a period of negative covariance, reflecting their negative impact on equity prices and positive influence on the short rate. By contrast, the covariance becomes more positive following a stock market shock, as the system is likely to be driven by additional stock market shocks, which generate movements in the interest rate in the same direction.

The covariances between the other two pairs of variables also exhibit rich patterns that differ across the various shocks considered. An important driver of the covariance between the short rate and the yield curve slope appears to be shocks to the short-term interest rate, reflecting the large negative impact of those shocks on the yield curve slope. The covariance between the yield curve slope and stock prices appears to be pushed down by shocks to both the short rate and the yield curve slope, largely because the volatility of yield curve slope shocks tends to remain elevated after these innovations.

Figures 2 and 3 highlight the most important implication of this model—that understanding the source of the shock driving asset price movements is crucial for accurately predicting the behavior of asset prices going forward. Analyzing the behavior of a single asset price in isolation could be misleading. Indeed, the exact same movement in the price of a given asset could be driven by an innovation to its own shock, or by the endogenous response to a shock to another asset price. Those two scenarios would have very different implications for the second moments of the variables going forward. By recovering the interactions between variables in equations (12) to (13), the model estimated above allows one to determine the source of the shock by looking at the contemporaneous movements in other asset prices. Once that source is identified, the implications for the behavior of asset prices going forward can be derived from the model's estimates.

The importance of the above considerations can be seen by the variation in the second moments of the financial variables over our sample. Figure 4 shows the evolution of the conditional variances of the structural-form innovations. The most noticeable features are the three episodes during which the variances of all three innovations spiked higher, which correspond to the 1987 stock market crash, the financial market turmoil in the fall of 1998, and near the year-end in 2000.[14] But the variances have some interesting patterns beyond those episodes. Most notably, the volatility of the equity price innovaton remained quite high for a period following the 1987 crash and, after moving to very low levels in the mid-1990s, has again become quite elevated over the past three years.

Figure 5 shows the movements in the conditional correlations between the financial variables generated by the changes in the variances of the underlying shocks. Each of the three correlations has varied over a sizable range during the past fifteen years. For example, the conditional correlation between equity prices and the short-term interest rate has been negative over most of the sample, but it has generally moved into positive territory over the

[14]The volatility in late 2000 may have partly reflected the anticipation of monetary policy actions, as the Federal Reserve began to ease on January 3, 2001. Asset price movements at that time might have been exacerbated by thin year-end liquidity conditions.

most recent three years. The primary reason for this pattern is that innovations to stock prices have been unusually volatile over that period, causing the positive reaction of the short-term interest rate to show through to the correlation between those variables.

The conditional correlation between the short rate and the yield curve slope instead remains negative over nearly all of the sample, although it went through a period in the mid-1990s when it moved closer to zero. That pattern primarily reflects that the relative importance of shocks to the yield curve slope increased at that time, in large part because of a decline in the volatilities of shocks to equity prices and the short rate. For the same reason, the conditional correlation between the yield curve slope and stock prices was strongly negative in the mid-1990s. This correlation has remained negative throughout the sample. However, it has moved towards zero in recent years as the relative importance of stock market shocks has increased.

Overall, the results demonstrate that the relative importance of financial market shocks has varied considerably since the mid-1980s, and that this variation has had important implications for the comovements between asset prices. Understanding and measuring this behavior is of obvious relevance to financial market participants in many different areas. In the next section we explore one application of this model, in a risk management exercise, which allows us to gauge the practical importance of these effects.

4 An Application to Risk Management

The previous section demonstrated that the conditional second moments of asset price movements vary considerably over time as the relative volatilities of the underlying shocks shift. This observation would presumably have important implications for forming portfolio decisions, managing risk, and pricing derivative securities. We demonstrate the practical importance of the model in a very simple risk-management application.

Consider the risk of a portfolio that is split evenly between the S&P 500 index and a ten-year Treasury note (which, for simplicity, we will assume to have a constant duration of $8\frac{1}{2}$ years). The portfolio suffers a 5 percent loss if equity prices fall by 10 percent or if the long-term Treasury yield increases by 125 basis points. We are interested in the amount by which an investor who does not properly account for contemporaneous interactions between asset prices would mismeasure the risk of this portfolio over time. We assume that this investor correctly measures the conditional variances of the three financial variables, but believes that the correlations between them are fixed at their unconditional values. We then calculate the percent by which the investor's estimate of the variance of the portfolio's daily returns exceeds the true variance from our model.

The top panel of Figure 6 shows the degree of risk mismeasurement in response to a two-standard-deviation innovation to each of the structural-form shocks. Following an equity price shock, the investor overestimates the portfolio risk by more than 3 percent. The reason is that he fails to realize that the shock induces a more positive correlation between the long-term yield and stock prices going forward (which arises, as discussed above, because the shock is followed by a period during which equity market shocks are more volatile). In that case, movements in the prices of his two assets tend to partly offset one another, so that the portfolio positions are better hedged than otherwise. The mismeasurement of risk tends to last about a month. By contrast, following a yield curve slope shock, the investor underestimates the risk of his portfolio by about 3 percent. This is because he fails to realize that stock prices and bond yields tend to move in opposite directions following the shock, thus adding to the risk of his portfolio. Lastly, the investor overestimates risk by about 2 percent following an innovation to the short rate, primarily because he overestimates the volatility of the long-term rate. Recall that we assumed the investor knows the true volatility of the yield curve slope. However, he fails to realize that the correlation between the short rate and the yield curve slope becomes more negative following the shock, which damps the true volatility of the long-term interest rate.

Table 2 shows that the risk mismeasurement immediately following each of the three structural shocks is considerable across a variety of portfolios. Of course, investors might be concerned about measuring risk over a longer period than one day, and the impact of the structural shocks would cumulate over time. Using the estimated variances of the structural-form innovations, we can compute the amount by which the investor, on average, would mismeasure the risk of holding his portfolio over a 30-day horizon. The results, shown in the last column, indicate that the (absolute value of the) imprecision of the investor's measurement of risk would typically be quite substantial. This finding is directly related to the extensive variation in the conditional correlations between the variables shown earlier in Figure 5.

Moreover, the risk measurement implications of failing to account for spillovers across financial variables would likely be even more severe than our example suggests. In the above exercise, we assumed that the investor was able to correctly measure the conditional variances of the variables. However, because he would not be able to recover the structural-form shocks that have the GARCH effects, the investor would likely mismeasure the conditional variances in addition to the conditional correlations.

Table 2: Risk Mismeasurement

Portfolio *(T-bills, Stocks, T-notes)*	Impact of: η^i	η^s	η^y	Ave. Abs. Value (30-day Return)
(0.00, 0.00, 1.00)	14.5	5.7	-3.6	22.5
(0.00, 0.25, 0.75)	8.4	5.5	-4.3	18.0
(0.00, 0.50, 0.50)	2.1	3.2	-3.1	11.5
(0.00, 0.75, 0.25)	-0.2	1.1	-1.3	5.4
(0.00, 1.00, 0.00)	0	0	0	0
(0.25, 0.00, 0.75)	14.5	5.7	-3.6	22.6
(0.25, 0.25, 0.50)	6.1	4.8	-4.2	16.0
(0.25, 0.50, 0.25)	0.1	1.7	-1.9	7.5
(0.25, 0.75, 0.00)	-0.1	0.0	-0.0	0.2
(0.50, 0.00, 0.50)	14.6	5.8	-3.6	22.6
(0.50, 0.25, 0.25)	2.2	3.2	-3.2	11.7
(0.50, 0.50, 0.00)	-0.2	0.0	-0.1	0.5
(0.75, 0.00, 0.25)	14.6	5.8	-3.6	22.5
(0.75, 0.25, 0.00)	-0.5	0.1	-0.2	1.5
(1.00, 0.00, 0.00)	0	0	0	0

Risk mismeasurement is the percent by which an investor who ignores spillovers would overestimate the variance of the portfolio shown.

Obviously, the above example is constructed to be very simple. More realistic risk management exercises might be concerned with imposing constraints on the total risk of a more complex portfolio or with calculating optimal hedging ratios for various positions. But even though actual risk management applications would be far more complicated, the basic implication of the model is clear—that measuring spillovers across financial markets is crucial to forming prudent risk management decisions.[15] The model developed here offers an approach for measuring those spillovers, with obvious applications to the calculation of portfolio risk or optimal hedging strategies. Moreover, it should also be clear that measuring these spillovers, and their effects on the variances and covariances of asset prices, has much broader applications to making effective portfolio decisions and pricing derivatives and other assets.

5 Conclusions

Overall, the model indicates that spillovers from one asset price to another constitute an important component of the behavior of the financial variables considered. By relying on conditional heteroskedasticity for identification, this paper has been able to quantify these effects—a result that has been notably absent in the previous literature. Having a measure of these spillovers is crucial for interpreting financial market developments and for predicting future values of the correlations between financial variables.

The model relies on the notion that the different financial variables considered are driven by orthogonal shocks and their responses to one another. This setup seems plausible given the economic interpretations offered above—that the shocks largely represent monetary policy shocks, shifts in the risk preferences of investors, and the revelation of information about future inflation. Of course, the model does not require these exact structural interpretations. The specification above simply requires that movements in the financial variables into a structure described by (1).

This specification imposes some assumptions that could be relaxed in future research. First, we have assumed that the markets depend linearly on each other, and that those coefficients do not depend on the underlying volatilities. Second, even though the model could be estimated including some common shocks, we have choosen not to do so here for

[15]For example, there appears to be growing interest in the ability to form hedging positions across different asset classes. Indeed, J.P. Morgan Securities (2002) writes that "In recent months, we have seen an explosion of interest in the use of equity and equity related instruments to hedge credit positions." Measuring variation in the correlation between equities and Treasuries would obviously be a critical step in this process.

simplicity. Third, the model have assumed a very simple GARCH structure, whereas the literature has refined the dynamics of volatility in a number of ways.

Despite these simplifications, the model seems to be quite effective at capturing singificant interactions between U.S. financial variables. Thus, we view the model as a useful first step in understanding the dynamics of these variables and the importance of spillovers between them. As should be clear, measuring these interactions has critical implications in a number of applications, ranging from risk management to option pricing.

Moreover, the paper serves to highlight a methodology that could be used in other empirical studies. A simple extension of this paper would be to include additional asset prices in the exercise. For example, one could investigate whether the exchange rate reacts more strongly to interest rate or equity price movements. An important consideration in this regard is whether some of the anomalous behavior of a subset of asset prices (such as the exchange rate and the interest rate) found in previous studies arises from the failure to capture their responses to other financial variables that have been omitted.

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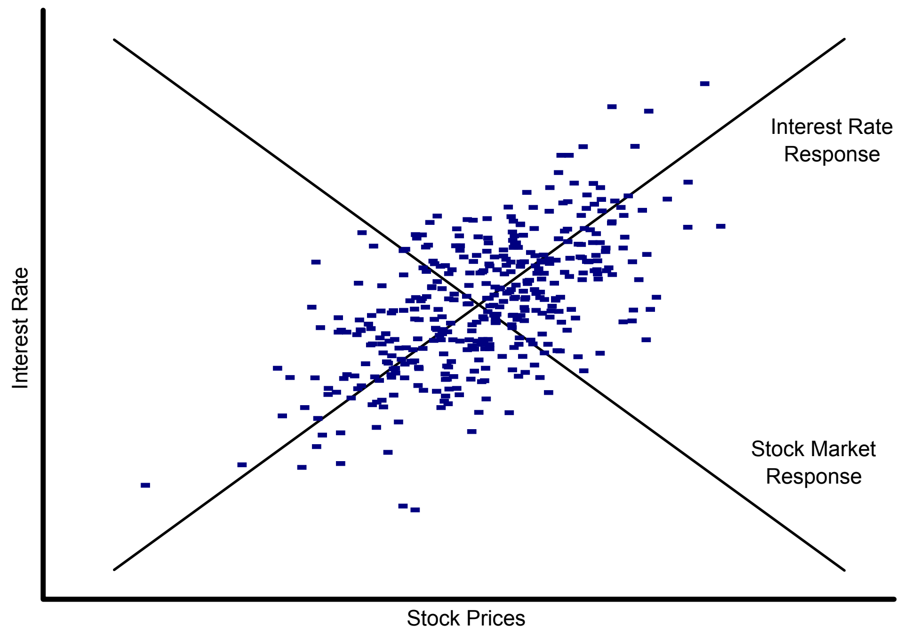
Figure 1
Joint Determination of Interest Rates and Stock Prices



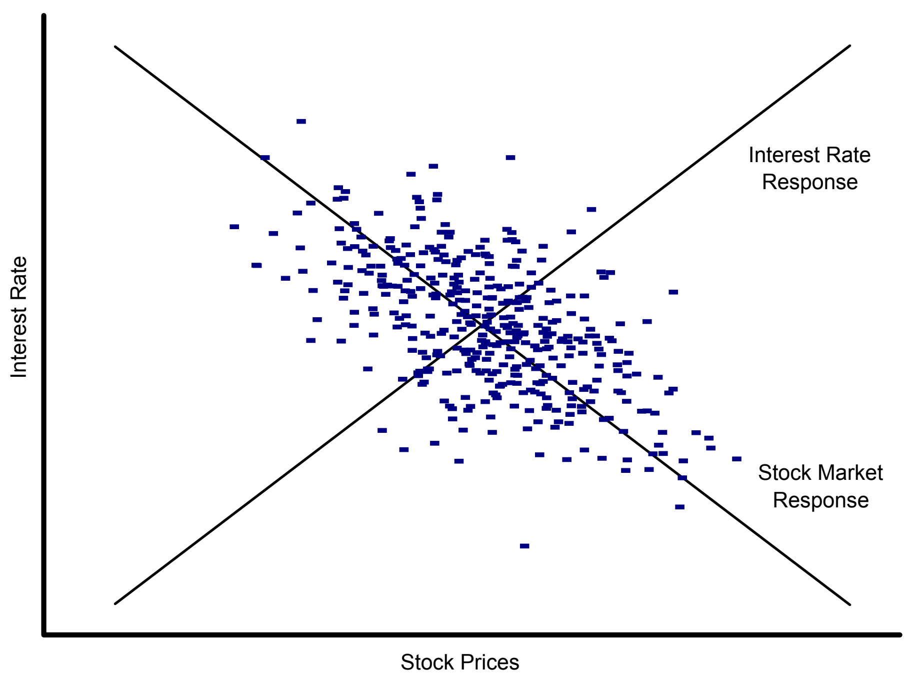
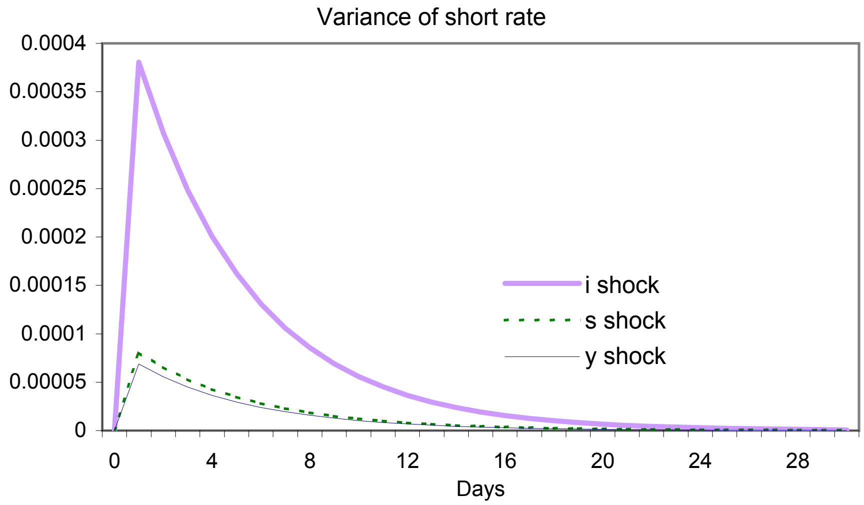
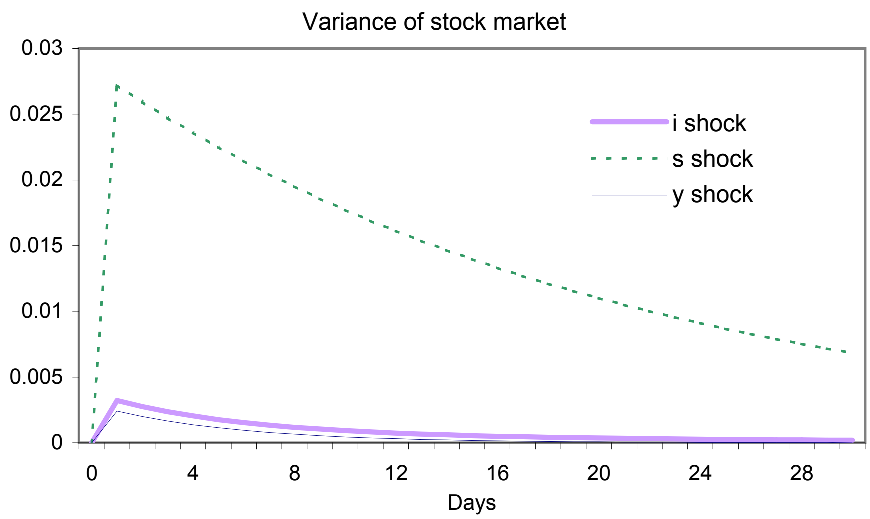


Figure 2:
Impulse Responses of Condiitonal Variances





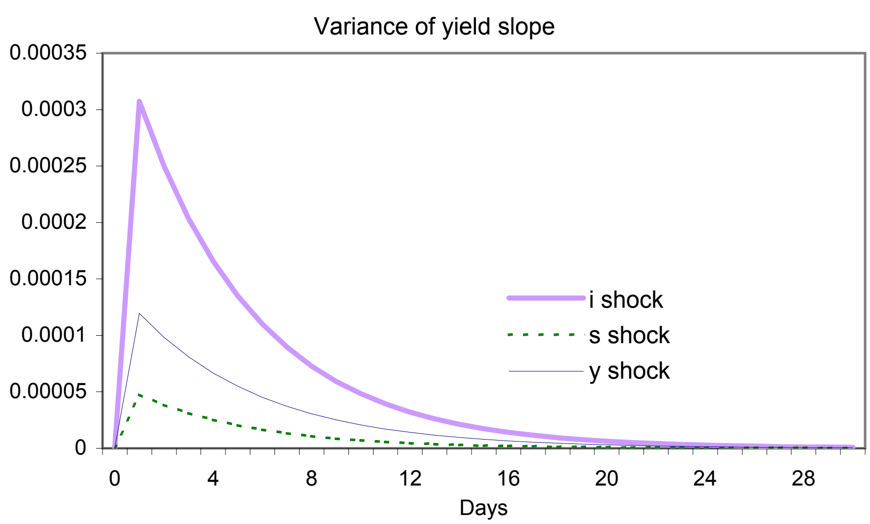
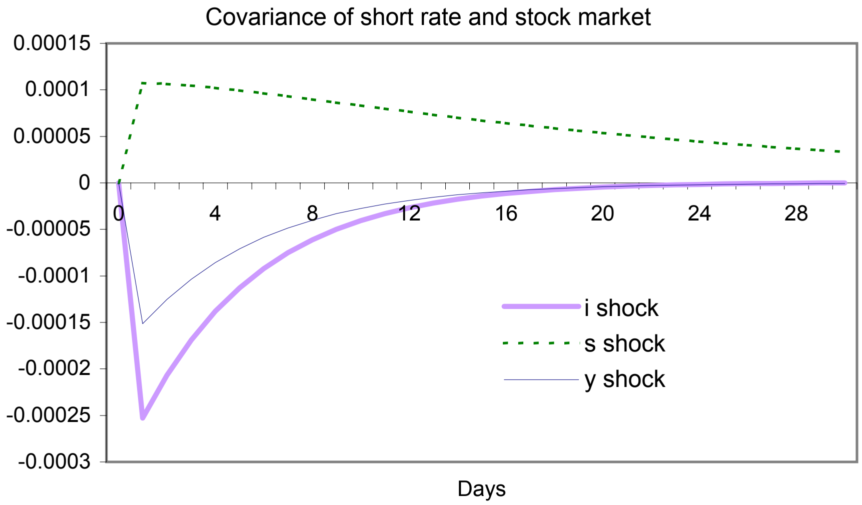
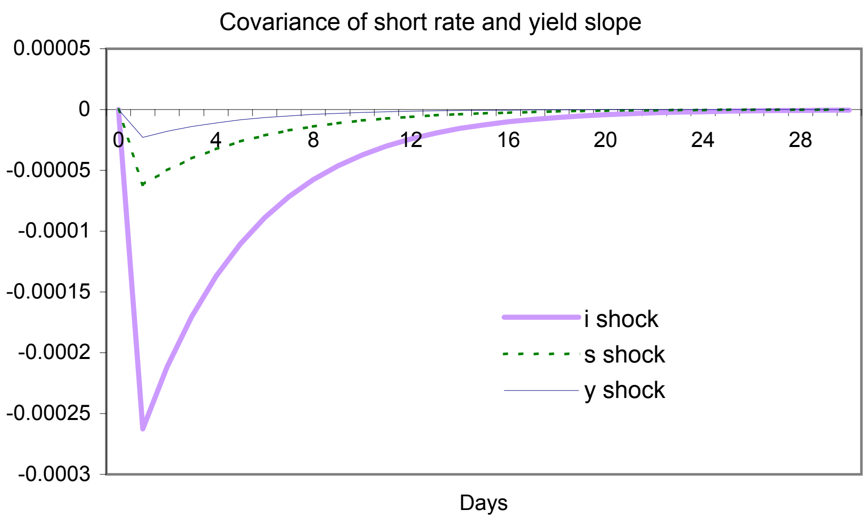


Figure 3:
Impulse Responses of Condiitonal Covariances





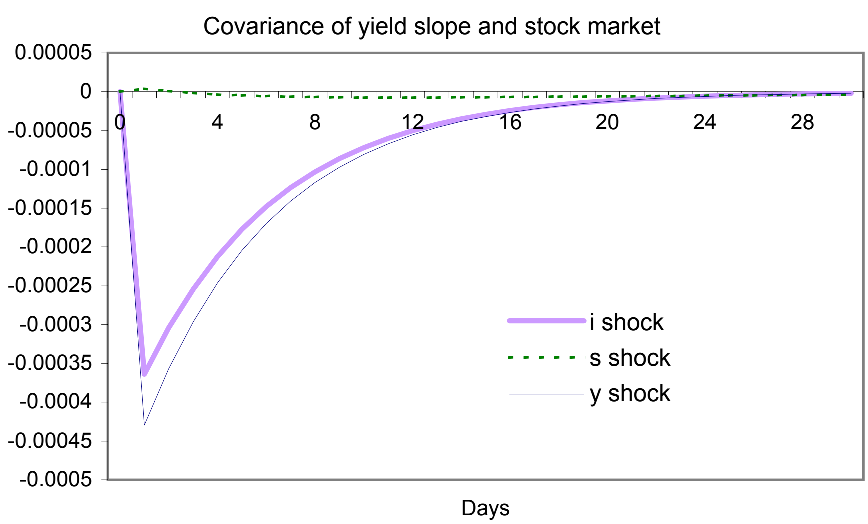
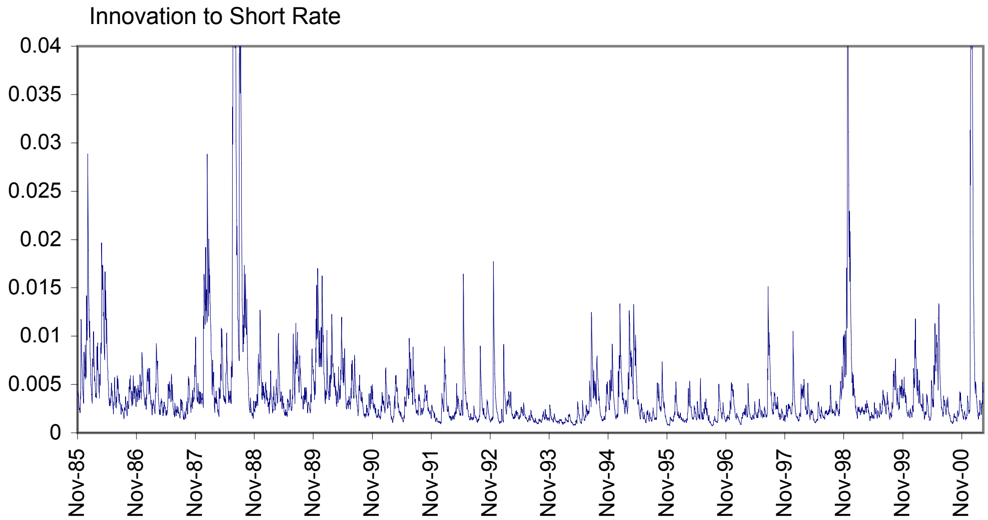
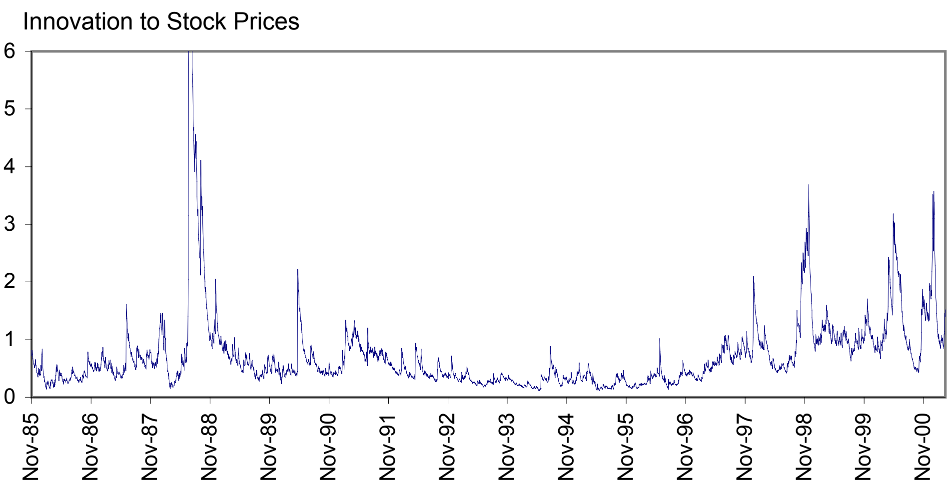


Figure 4: Conditional Variances of Structural-form Innovations





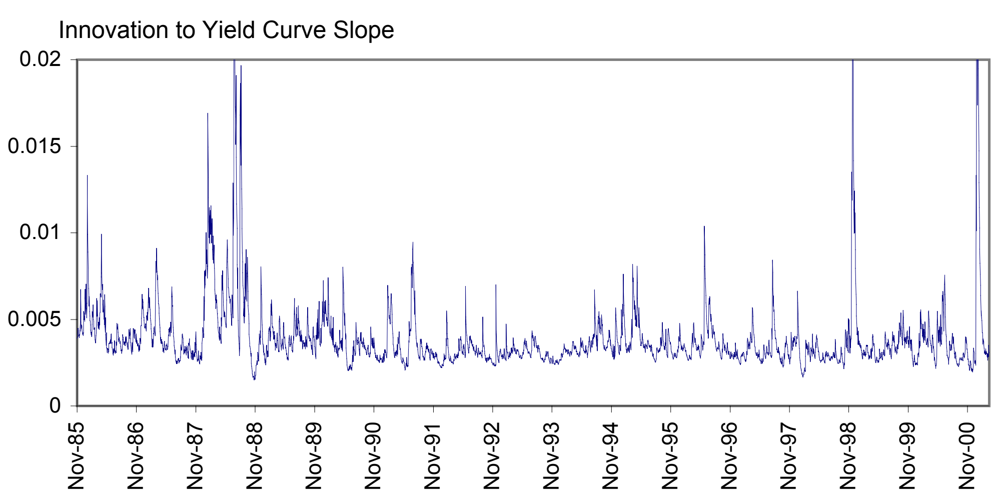


Figure 5: Conditional Correlations between Financial Variables

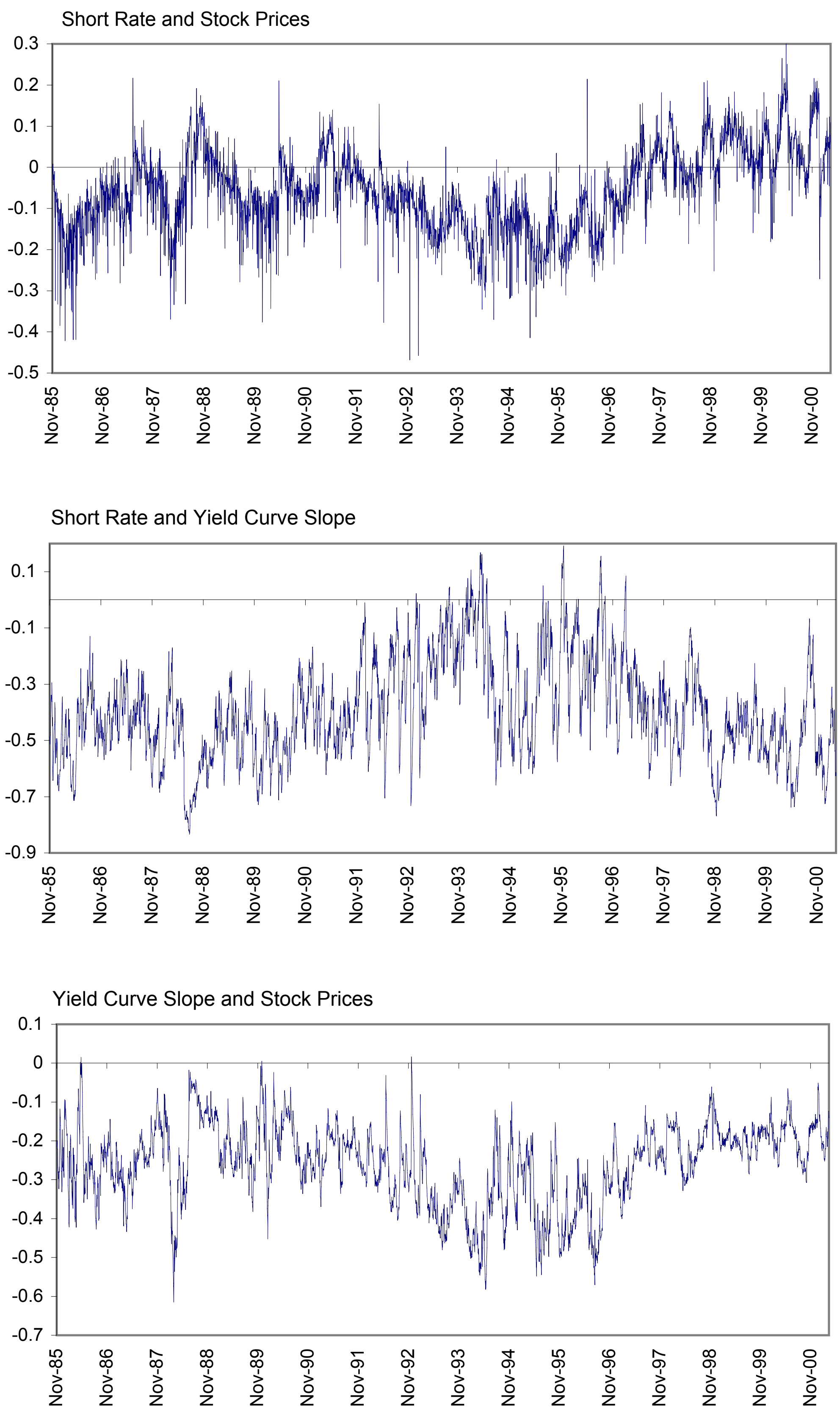


Figure 6:
Impulse Responses of Mismeasurement of Portfolio Risk

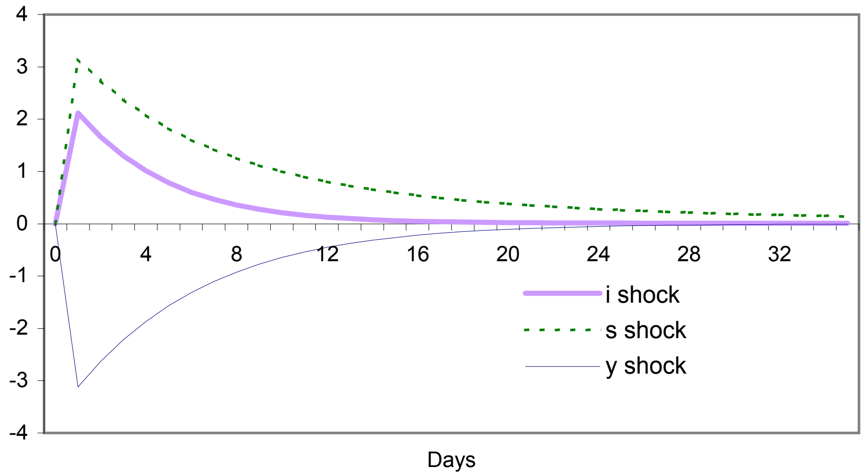


Figure shows the percent by which the investor who ignores spillovers would overestimate the variance of a portfolio evenly split between the S&P 500 and the ten-year Treasury note.