Repo Markets, Counterparty Risk, and the 2007/2008 Liquidity Crisis*

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Abstract. A standard repurchase agreement between two counterparties is considered to examine the endogenous choice of collateral, the feasibility of secured lending, and welfare implications of the central bank's collateral framework. As an innovation, we allow for two-sided counterparty risk. In line with empirical observations, it is shown that the most liquid and least risky assets are used as collateral in market transactions first. An endogenous opportunity cost arises from using liquid collateral with the central bank. Conditions are identified such that expected utility increases for all market participants when the central bank accepts a broader range of assets as collateral. *JEL classification:* G21, E51.

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Introduction

Standard (sale and) repurchase agreements, or repos (RPs) in short, are used by both private and public counterparties to conveniently swap cash against collateral for a pre-specified period of time. In a typical contract, the lender of cash is compensated by an interest that is calculated from the nominal value of the transaction, the term, and the so-called repo rate. Moreover, a haircut is applied to the collateral to limit the lender's exposure to counterparty risk. Indeed, the lender faces the combined risk that the borrower is unable to repay principal amount plus interest, and at the same time the liquidation value of the collateral falls short of the lender's claim. Putting up more collateral keeps this risk contained.[1]

The theoretical analysis of repurchase agreements started with with a seminal contribution by Duffie (1996) who pointed out that when owners of a specific asset incur frictional costs from using the asset as collateral, the repo rate for the asset may fall significantly below the repo rate charged for general collateral. Moreover, through its impact on funding conditions, such specialness is predicted to add a premium to the asset's market price. In a number of recent papers, this theoretical prediction on competitive repo markets has been empirically confirmed from different perspectives.[2]

[1] Over the last few years, the repo segment has gained considerable importance in international money markets. For instance, daily turnover in the euro repo market has approximately doubled between 2002 and 2007, while the unsecured market segment has been expanding only moderately over the same period (cf. ECB, 2007a). For the U.S. market, Demiralp et al. (2006, p. 71) write that "(t)he overall repo market is reportedly far larger than the market for federal funds and overnight interbank Eurodollars." Generally, the markets in the U.S. and in the euro area share many commonalities, but there are also differences. For instance, the euro area repo market appears to be characterized by a higher number of institutional players, and by a somewhat more fragmented clearing and settlement landscape. Moreover, the U.S. market is concentrated on the overnight segment, while in the euro area, terms between overnight and one year are all common. The growth of international repo markets can be attributed to a wide range of factors including an increasing reliance on innovative strategies of funding and leveraging (e.g., by hedge funds), benefits from banking regulation such as capital adequacy (Basle II), a high degree of standardization in the contract documentation, and a prominent role of the instrument in central banks' implementation frameworks. The wider use of the repo instrument is also reflected in an increasing academic interest in national markets. See for instance papers by Baba and Inamura (2004), Fan and Zhang (2007), Jordan and Kugler (2004), and Wetherilt (2003).

[2] Jordan and Jordan (1997) validate specialness in repo rates using daily data for the U.S. Treasury repo market of overnight general collateral rates and special financing rates. Based on a data set for the German money market, Buraschi and Menini (2002) reject the rational expectations hypothesis of the term structure for the repo market, and find empirical evidence for a time-varying liquidity risk premium. Krishnamurthy (2002) identifies repo specialness as a cost-of-carry that renders convergence trading much less profitable than suggested by bond spreads.

An assumption underlying this existing theory of the repo market is that there is an investor (the "Short") who seeks to get hold of a well-specified asset through the repo market transaction. However, it has been noted at various places that repo markets are generally open not only to investors interested in a specific security, but also to investors that are interested primarily in the cash side of the transaction. That is, there are also repurchase agreements that are driven mainly by the funding/deposit motive, so that the choice of collateral becomes part of the negotiation.[3] As a practical matter, the difference in the motive for approaching the market not only needs to be revealed early in the negotiation, but is also reflected in differences in the margining (either in cash or in collateral). Moreover, in the case of cash-driven repos, the repo rate for less liquid collateral may also exceed the rate for general collateral.[4] The present paper aims at exploring the determinants of collateral in such cash-driven repurchase agreements. To this end, we introduce counterparty risk into a partial equilibrium model of bilaterally negotiated repurchase agreements.

Two empirical regularities have motivated this route of inquiry. One observation is that typically only collateral of the highest quality is accepted in the interbank market.

This can be seen by comparing the collateral used in the private repo market with the collateral used in repo auctions conducted by central banks. For instance, as shown in Table I, the collateral used during 2006 in the private euro repo market has been mostly government bonds. Illiquid and risky assets such as asset-backed securities (ABS) are *not* commonly employed as collateral in the private bilateral repo market. This situation stands in stark contrast with the composition of collateral held with the European Central Bank (ECB) that accepts a wide range of assets including government bonds (issued either by central or regional authorities), bank bonds (both uncovered and covered), corporate bonds, ABS, other marketable securities, and credit claims. In fact, during 2006, only about 29 percent of assets

Table I about here

[3]As far as we know, there is so far no empirical evidence on the share of interbank repo transactions in the euro area that is cash-driven. For instance, in Comotto's (2007, p. 17) product analysis, there is no split-up for the bulk of the repo desk activity. However, evidence for the U.S. market surveyed by Buraschi and Menini (2002, p. 253) suggests a role for the funding motive in repo markets even under normal market conditions.

[4]For instance, Griffiths and Winters (1997) document an average spread for repos on collateral issued by the Government National Mortgage Association (GNMA) over government bonds of 8.5 basis points during the period February 1984 through January 1985.

deposited for use as collateral in Eurosystem credit operations were issued by governments. As Table I indicates, the bulk of central bank collateral in the euro area has been composed of less liquid asset types such as uncovered bank bonds, covered bank bonds, and asset-backed securities.

The second regularity in the data relates to more recent developments. Specifically, following the summer 2007 financial market turbulences, requirements on collateral assets imposed by cash lenders in the interbank market became even stricter than they usually are. For instance, Frediani et al. (2007, pp. 15-16) report that the share of structured securities used as collateral in so-called tri-party repos has fallen from 35 percent to 25 percent between June 1 and September 14, 2007, with ABS Auto, Card, CDOs, and MBS the most affected through the subprime crisis.[5] This is consistent with observations by Comotto (2008, p. 19) who writes that "Concern over the quality of collateral could explain the reduction in the share of tri-party repos, which has been the preferred way of managing non-government collateral. It definitely explains [...] the unusually high share of government bond collateral in tri-party repos." In contrast, the composition of central bank collateral has shown just the opposite development. Indeed, media reports suggest that the share of illiquid and relatively risky assets such asset-backed securities has increased significantly since the beginning of the turbulences in August 2007.[6]

To better understand these observations, the present paper takes a closer look at the role of collateral in interbank lending relationships. A hypothetical scenario is studied in which two counterparties, a cash borrower and a cash lender, negotiate simultaneously about (a) the collateral assets to be used, (b) the haircut, and (c) the repo rate. Extending the existing theoretical framework, we allow for two-sided counterparty risk, i.e., the possibility that the borrower of cash and likewise the

[5] In a tri-party repo, counterparties sign an additional agreement with a so-called tri-party agent, who essentially serves as a custodian for the collateral assets. Compared to a bilateral repo, the administrative burden of handling collateral is significantly reduced for the counterparties. Tri-party repos are still more common in the U.S. than in the euro area. In the U.S., the tri-party segment corresponds to very roughly one quarter of the repo market, in the euro area, the share has typically been about one tenth of total market turnover.

[6] Similar developments have been observed in U.S. open market operations. For instance, the Federal Reserve Bank of New York (2008, p. 21) writes that "in recent years the distribution by collateral tranche of outstanding RPs has been weighted heavily toward the Treasury tranche...until financial market strains appeared in short-term funding markets. At that point dealers' propositions against agency and MBS collateral tranches that it accepts on its RPs became more attractive on a relative basis."

lender of cash may default. This has potentially important consequences for the economic determinants of collateral. The analysis will also enable us to formally discuss welfare consequences of the central bank's collateral policy.

With two-sided credit risk, the bilateral negotiation can be expected to lead to an agreement that balances financial benefits and risks on both sides of the transaction. Specifically, the cash lender may be willing to accept a somewhat lower repo rate in exchange for a somewhat increased haircut, as a higher haircut implies better protection. Conversely, the cash borrower may be willing to provide somewhat more collateral for a somewhat lowered repo rate. This is not costless, however, because there is the real risk that collateral deposited by the cash borrower may get lost in the cash lender's insolvency mass.[7] An efficient bargaining outcome is achieved, therefore, by making the marginal rate of substitution between haircut and repo rate congruent between the two counterparties. It turns out that, if collateral is not perfect, i.e., if price fluctuation or illiquidity are possible, then it is typically efficient to expose both parties to non-trivial counterparty risk.

This unavoidability of counterparty risk is what ultimately drives our first main result. This result says that if two counterparties agree to transact, they typically agree to use the most liquid and the least risky assets of the borrower as collateral first. Thus, in a bilateral transaction between two counterparties that may each default with positive probability, good collateral drives bad collateral out of circulation, suggesting an intuitive comparison with Gresham's law for commodity money.

We go on and study the general feasibility of secured contracting under market stress. It is shown that if the most liquid and least risky assets of the borrower are still relatively illiquid or risky then the two counterparties may, even under symmetric information and zero opportunity costs of collateral, not be able to agree on

[7]The institutional literature has repeatedly stressed this issue. For instance, Stigum (1989, p. 325) writes that "Sophisticated managers of large bond portfolios exercise extreme care in determining to whom they will reverse out their *valued* bonds" (emphasis in the original). Corrigan and de Terán (2007, p. 76) emphasize the same point: "It is often mistakenly thought that the provider of cash has the greater credit risk but this is not necessarily so." A case illustrating the symmetric nature of counterparty risk in collateralized transactions is the failure of the securities dealer Drysdale in 1982. According to Garbade (2006, p. 32), "it was quickly evident that firms that had lent securities to Drysdale were inadequately margined and were going to be left with far less cash than the replacement cost of their securities." For an illustration within our theoretical framework, see the caption to Figure 2.

a transaction at all. This outcome occurs in particular if default probabilities are perceived as non-negligible by market participants, which might relate our analysis to the developments in money markets following early August 2007. The imperfection of the repo market under two-sided credit risk also adds to existing structural explanations of the microstructure of the money market based on asymmetric information, and explains the existence of central counterparties. Last but not least, this second result allows us to apply a theoretical argument that has been put forward recently by Kashyap et al. (2002).

The final part of the analysis explores the question of how the central bank's collateral policy might affect overall welfare. It is shown that the expansion of the set of collateral eligible for central bank operations may lead to a welfare improvement for market participants. However, as we also show, the expansion of the set of eligible collateral is typically accompanied by a replacement of liquid collateral by illiquid collateral in the primary market. I.e., in contrast to the prediction obtained for market transactions, bad collateral drives out good collateral in lending relationships with the central bank. More generally, the model allows discussing the collateral frameworks of central banks both in the context of fiscal discipline of euro area member countries and in the context of the subprime crisis.

The analysis relates to further strands of the theoretical literature. One is concerned with credit rationing and collateral under one-sided credit risk. Stiglitz and Weiss (1981) have shown that credit rationing may occur as a consequence of asymmetric information either at a pre- or post-contracting stage. Bester (1985) has argued that in the case of pre-contracting asymmetric information, the self-selection problem may be resolved when commitment to costly collateral is feasible for entrepreneurs with relatively low risks.[8] Hellwig (1987) has argued that it may be hard to decide whether collateral may serve as an effective sorting device in a given credit market, because the nature of the refined equilibrium will typically depend on the way in which competition is modeled under adverse selection. Berger and Udell (1990) even conclude that existing theoretical and empirical approaches to the use of collateral still have to be reconciled. A potential solution has recently been

[8]See also Chan and Kanatas (1985), Chan et al. (1986), Besanko and Thakor (1987a, 1987b), and Chan and Thakor (1987).

offered by Coco (1999).

As an alternative to this self-selection view on the use of collateral, models have been developed that assign a key role to moral hazard and observable risk characteristics. Boot et al. (1991) developed an approach based on the idea that collateral mitigates moral hazard on the side of the borrower, so that borrower risk is positively correlated with collateral usage. Manove and Padilla (1999, 2001) argue that collateral induces banks to do less careful screening of loan applicants. Based on a Spanish data set, Jiménez and Saurina (2004) come to the conclusion that the incentive view explains the use of collateral better than the self-selection view. More recently, Jiménez et al. (2006) have found support for the hypothesis that, for the case of business credit, observable risks matter in decisions about collateral. Booth and Booth (2006) likewise conclude that collateral pledges are correlated with riskier loans. Inderst and Mueller (2007) show that collateral can help to resolve an inefficiency in credit markets with imperfect competition.[9]

The rest of the paper is structured as follows. Section I introduces the model and discusses efficient risk sharing in standard repurchase agreements involving two-sided credit risk. In Section II, we describe a simple effect that might explain why interbank transactions rely predominantly on liquid collateral. Section III studies the possibility of repo market disruptions. The residual nature of central bank collateral is formally described in Section IV. Section V elaborates on central bank policy and welfare consequences. Section VI concludes. All proofs are relegated to the Appendix.

I. The basic model

Consider a money market over three dates, date 0, date 1, and a terminal date 2. There are altogether $1 + m$ assets: cash and $m \geq 1$ collateral assets $j = 1, ..., m$. Cash is riskless and does not carry interest. Collateral assets may be either risky or illiquid or both, as will be made more precise later. There are two counterparties

[9] Still another strand of literature related to the present study is concerned with rediscounting and payments. Freeman (1996) considers a model with overlapping generations in which fiat money is used both for consumption and for repayment of loans. It is shown that an elastic provision of liquidity within the period can resolve temporary tensions in liquidity demand without affecting price levels for the consumption good. Mills (2006) considers liquidity provision from a mechanism design perspective, and shows in particular that distortions may occur when the central bank requires collateral that offers alternative benefits for borrowers in the economy.

$i = 1, 2$ in the market. We will think of these as commercial banks, but with minor changes to the interpretation, the analysis should apply also to other financial and non-financial institutions.

Each of the two banks has an exogenous initial endowment of cash and collateral assets at date 0. Moreover, each bank is required to hold a certain amount of cash (potentially zero) at the end of date 1. Cash held in excess of these minimum reserve requirements will be of no value, i.e., there is no carry-over provision. Moreover, initial endowments in cash are such that reserve requirements would be fulfilled without slack in the absence of further transactions.

The time structure of the model is shown in Figure 1. Between dates 0 and 1, there is a publicly observable random customer request to transfer an amount $\lambda > 0$ of cash at date 1. With equal probability, the transfer will be from Bank 1 to Bank 2 or vice versa from Bank 2 to Bank 1. The absolute size λ of the liquidity shock may also be random. However, without loss of generality, λ will initially be normalized to one "unit." To compensate for the liquidity shock, the bank receiving the transfer, bank i_L, will seek to become the *lender* (of cash) in the money market, while the bank sending the funds, bank i_B, will seek to become the *borrower* (of cash).

Figure 1 about here

By definition, if not defaulted, a commercial bank in the role of the borrower (lender) is equipped at date 2 with sufficient assets to repay principal amount and interest (to redeliver the collateral). Without loss of generality, there are then three states of nature: In the "good" state G, neither the lender nor the borrower defaults; in state B, only the borrower defaults; and in state L, only the lender defaults. Denote by $\pi_\omega = \pi_\omega(i_B, i_L)$ the probability that state ω realizes at date 2, where $\omega \in \{G, B, L\}$. Clearly, $\pi_G + \pi_B + \pi_L = 1$.[10]

The following assumption is fundamental to all what follows. To our knowledge,

[10] In general, default probabilities might depend also on the terms of the interbank transaction. We have excluded this possibility for the sake of simplicity. Note, however, that endogeneity need not affect the economic determinants of collateral. For instance, imagine a scenario such that a contract with a marginally higher repo rate marginally increases (decreases) the probability of the borrower's (lender's) default. In this scenario, an improved composition of the collateral should still engender a Pareto improvement for borrower and lender. The situation is more complicated if default probabilities may depend on either haircut or collateral composition. However, intuition suggests that also in this case, and provided that default probabilities of either counterparty are weakly decreasing following a reduction in the exposure from the transaction, the mutually beneficial effect of using higher quality collateral should remain. Thus, we would expect that introducing endogeneity would affect only equilibrium values for repo rate and haircut, but not the optimal composition of collateral.

it also marks the departure from the existing theoretical literature on collateralized lending (in addition to the references given at the end of the introduction, see, e.g., Barro, 1976, Benjamin, 1978, Plaut, 1985, Cossin and Hricko, 2003).

Assumption 1. (Two-sided credit risk) $\pi_B > 0$, $\pi_L > 0$.

To mitigate two-sided credit risks, banks might in principle want to write complicated contracts that condition on all the information observable and verifiable at date 2. However, to make progress, we shall instead consider an institutional form of the repo contract.[11] Specifically, it is assumed that counterparties may sign a *standard repurchase agreement (SRA)* $C = (y, h, r)$, which is composed of a collateral composition y, a haircut $h \geq -1$, and a repo rate r. Here and later on, a *composition* is a collection $y = (y_1, ..., y_m)$ of weights $y_j \geq 0$ for individual assets $j = 1, ..., m$ such that $\sum_{j=1}^{m} y_j = 1$. The border case $h = -1$ will correspond to an unsecured loan. The agreement foresees that the lender promises to transfer one unit of cash at date 1. The borrower in turn promises to deposit collateral of composition y with the lender at date 1.[12] Moreover, the common haircut h is applied to all assets.[13] At date 2, in the good state, the borrower will repay the principal amount plus an interest (rate) r. The lender, in turn, redelivers the collateral to the borrower.

So far, no provisions have been made to cover counterparty risk. For instance, should the borrower default, the lender would have no legal basis for liquidating the collateral asset. Indeed, as the interbank contract matures, the lender's claim on repayment of principal and interest will meet the borrower's non-monetary claim

[11] Indeed, in practice, repurchase agreements are highly standardized on an international level (cf. Garbade, 2006, and Comotto, 2007). Standards are set by the Securities Industry and Financial Markets Association (sifma), which – at the time of writing – is updating the International Capital Market Association's contract document, the so-called TBMA/ISMA Global Master Repurchase Agreement (cf. The Bond Market Association, 2000). For an analysis of collateralized debt that is not imposing a standardization assumption, see Lacker (2001).

[12] In reality, the lender of cash may grant the borrower the right to substitute collateral by assets of equal or better quality at any time during the term of the transaction. Indeed, a preference for flexibility on the side of the borrower should allow the lender in a cash-driven repo to extract a higher rent by offering the substitution right. In the basic model, however, such discretion would be of little value for the borrower, that is why it is not part of the negotiation. In Sections IV and V, we extend the basic model so that the cash borrower may substitute collateral that had been placed with the central bank.

[13] Equivalently, but more demanding in terms of notation, the contract could specify an individual haircut for each collateral asset used in the transaction, where the collateral composition should be adjusted correspondingly. Note that haircuts are fixed during the term of the repurchase agreement. For a dynamic model of lending with haircuts that are linked to historical value-at-risk figures, see Brunnermeier and Pedersen (2008).

on the collateral. In the worst case, a "cherry-picking" insolvency agent of the defaulting borrower would refuse payment, while demanding delivery of the collateral! Likewise, without provisions, the borrower would have no right to withhold repayment of principal and interest when the lender does not return the collateral. The institutional solution of this problem is to allow for setting-off (or netting) of mutual claims in case of insolvency of one counterparty.

Note that any netting rule must transform the borrower's claim for delivery of the collateral into a monetary claim. Following legal practice, we will assume here that the size of the monetary claim is determined by market conditions at the time of default.[14] Let $\widetilde{v}_b$ (and $\widetilde{v}_a$) denote the liquidation value (replacement cost) of the collateral portfolio at date 2, conditional on the borrower's (lender's) default, with realized value v_b (and v_a).[15]

Assumption 2. (Netting) *In state B, the borrower's claim on the collateral is replaced by a claim of payment of v_b. In state L, the borrower's claim is replaced by a claim of payment of v_a. Subsequently, the claim of the non-defaulting party vis-à-vis the defaulting party may be used to set off the claim of the defaulting party vis-à-vis the non-defaulting party.*

For instance, in state B, the lender's claim on repayment of principal plus interest is protected by the collateral only if the realized liquidation value v_b of the collateral portfolio at date 2 covers $1 + r$. Thus, the lender incurs a potential loss of $\min\{v_b - (1 + r); 0\} \leq 0$ compared to state G. Similarly, in state L, the borrower has a potential loss of $\min\{(1 + r) - v_a; 0\} \leq 0$, where v_a is the realized replacement cost of the collateral portfolio at date 2. In reality, the extent to which such a potential loss becomes an actual loss depends on several factors including whether the insolvency assets of the defaulting party have some market value, and whether the net claim of the non-defaulting party is senior to claims held by other parties.

[14] The TBMA/ISMA Global Master Repurchase Agreement foresees a set-off of mutual claims in case of one-sided insolvency, where the non-defaulting party valuates collateral claims either by actual, quoted, or estimated market prices. Bliss and Kaufman (2006) offer an insightful discussion of netting provisions in the related case of derivatives.

[15] Alternatively, there is no market available at date 2, and prices reflect the respective second-best alternative. For instance, when no buyer can be found for the collateral, then $\widetilde{v}_b$ should be replaced by the risk-adjusted present value of the cash flow generated for the lender by holding the collateral until maturity net of costs of funding, all projected conditional on the borrower's default.

The following assumption is made for simplicity, and can be relaxed significantly without affecting our results.[16]

Assumption 3. (Subordination) *Any positive net claim of the non-defaulting party vis-à-vis the defaulting party will be completely lost.*

As an additional matter, the agreement must be specific about what happens when the defaulting party holds a gross claim that exceeds the claim of the non-defaulting party. In this case, the non-defaulting party would apparently like to walk away from the contract, terminating the relationship on a "no-fault" basis. For instance, in state B, the lender would want to sell the collateral and keep haircuts plus any potential interim increase in the market price. Similarly, in state L, the borrower would want to profit from a decline in the collateral value. Such surprise profits are not foreseen in the standard documents used in money market transactions.

Assumption 4. (No windfall profits) *If the defaulting party has a positive net claim vis-à-vis the non-defaulting party then the non-defaulting party has the obligation to pay the net claim (to the insolvency agent of the defaulting party).*

The specific form of Assumption 4 is motivated by current market practice. It is by no means necessary for our results. Indeed, it is not difficult to check that the subsequently derived theorems generalize into a setting where a positive share of windfall profits can be realized, provided that the share is weakly declining in these profits. Such an extension would also include the limit case where windfall profits could be fully kept by the non-defaulting party. However, for the sake of simplicity, we shall stick to Assumption 4 in the sequel.

Collectively, Assumptions 2 through 4 make the contract comprehensive and thereby determine conditional payoffs. Denote bank i's utility function by $u_i(.)$, for $i = 1, 2$. The function $u_i(.)$ is assumed to be twice continuously differentiable with $u_i'(.) > 0$ and $u_i''(.) \leq 0$. Each bank $i = 1, 2$ maximizes expected utility from terminal payoffs, where the utility in case of own default is normalized to zero. Utility is derived from the instrument's isolated short-term return, with variations in the valuations of collateral assets being of a temporary nature. In particular, the borrower experiences no changes in utility from short-term increases to the value of

[16] In particular, our results should remain valid if the share of the net claim actually lost by the non-defaulting party is always strictly positive and weakly increasing in the net claim.

collateral assets unless the lender defaults during the term of the repo, in which case the collateral needs to be replaced on short notice.[17]

Write $u_L(.) = u_{i_L}(.)$ and $u_B(.) = u_{i_B}(.)$. Let $\widetilde{u}_L$ and $\widetilde{u}_B$, respectively, denote the lender's and borrower's uncertain utility at the time of contracting. Then the lender's expected utility at the time of contracting is given by

$$E[\widetilde{u}_L] = \pi_G u_L(r) + \pi_B E[u_L(\min\{\widetilde{v}_b - 1; r\})], \tag{1}$$

where $E[.]$ denotes the unconditional expectation operator, and the minimum takes care of Assumption 4. Thus, conditional on the borrower's default, the lender is basically (i.e., ignoring the bid-ask spread) exposed to a short European put option on the collateral, where the strike price is determined by the degree of overcollateralization, i.e., by the haircut. Similarly, the borrower's expected utility at the time of contracting reads

$$E[\widetilde{u}_B] = \pi_G u_B(-r) + \pi_L E[u_B(\min\{1 - \widetilde{v}_a; -r\})], \tag{2}$$

which amounts to being exposed to a short European call option on the collateral, conditional on the lender's default. Note that in contrast to a vulnerable option (see Johnson and Stulz, 1987) that loses the option character in the default case, a repurchase agreement only transforms into an option *with* the default of either counterparty.[18]

A scenario will be considered now in which lender and borrower bargain to an efficient outcome. Let $q_j^i \geq 0$ denote bank i's initial endowment of collateral asset j, for $i = 1, 2$ and $j = 1, ..., m$. Apparently, the bargaining set for borrower and lender will consist of all standard repurchase agreements (y, h, r) that satisfy

$$y_j(1 + h) \leq q_j^{i_B} \qquad (j = 1, ..., m). \tag{3}$$

[17] It is this focus on liquidity risk that will lead to different expressions for expected utilities compared to what one would obtain in Merton's (1974) option pricing approach to collateralized lending.

[18] In general, counterparties' actual returns may differ from expressions given in (1) and (2) as a consequence of accounting rules. For instance, Griffiths and Winters (1997, p. 819) report that in the U.S., government and agency repos do not affect required reserves for a depository institution, whereas private-issue repos are exempt from Federal Reserve Board Regulation D. This might imply an indirect cost of using private-issue collateral in the U.S. repo market. The effect is absent, however, in the euro area because the Eurosystem generally applies a zero reserve ratio to all repo liabilities (cf. ECB 2005, p. 57).

An SRA (y, h, r) that satisfies (3) will be called *valid*. An SRA will be called a *true* SRA if $h > -1$ and $r > -1$. We will say that a valid SRA (y, h, r) *does not exploit* the borrower's collateral if there is a $h' > h$ such that (y, h', r) is valid. A valid SRA (y^*, h^*, r^*) is *efficient* when the pair of counterparties' expected utilities resulting from the contract is not dominated, in the Pareto sense, by expected utilities resulting from any other valid SRA. We will study now in some detail the properties of efficient repurchase agreements.

It is immediate that two-sided credit risk could be effectively eliminated if collateral assets would share the desirable properties of cash in terms of risklessness and liquidity. Typically, however, collateral is neither riskless nor perfectly liquid. So let $\widetilde{p}_b^j$ (and $\widetilde{p}_a^j$) denote the uncertain liquidation value (replacement cost) of asset j at date 2, conditional on the borrower's (lender's) default, with realized value p_b^j (and p_a^j). In organized markets, for instance, these figures would correspond to conditional bid and ask prices of individual collateral assets. The respective multivariate distributions of the vectors $(\widetilde{p}_b^1, ..., \widetilde{p}_b^m)$ and $(\widetilde{p}_a^1, ..., \widetilde{p}_a^m)$ on $\mathbb{R}_{\geq 0}^m$ are assumed to be common knowledge among market participants. For a given composition $y = (y_1, ..., y_m)$, let $\widetilde{p}_b = \sum_{j=1}^m y_j \widetilde{p}_b^j$ and $\widetilde{p}_a = \sum_{j=1}^m y_j \widetilde{p}_a^j$ denote the conditional liquidation value and replacement cost of the collateral portfolio net of haircuts, with respective realizations p_b and p_a. Then, best-practice accounting of the haircut in repurchase agreements implies $v_b = (1 + h)p_b$ and $v_a = (1 + h)p_a$, where market valuations at the value date correspond to "dirty" prices which include accrued interests on the collateral securities during the term of the repo.[19]

It turns out that an efficient SRA will typically expose both lender and borrower to non-trivial counterparty risk. To make this statement precise, the following definitions turn out to be useful. For a given collateral composition y, let $\underline{p}_b = \underline{p}_b(y)$ denote the minimum of the support of $\widetilde{p}_b$ and $\overline{p}_a = \overline{p}_a(y)$ the supremum of the support of $\widetilde{p}_a$. We will say that collateral is *imperfect* if $\overline{p}_a > \underline{p}_b$ for any y. Furthermore, we will say that collateral is *insured* if there is some composition y such that either

[19]Our analysis does not presuppose marketability of collateral assets at the time of contracting. However, there is one interpretation of the model in which all collateral assets are perfectly liquid at the time of contracting and possess a market price of 1 at that stage. Note also that if collateral assets are assumed to be marketable both at the time of contracting and in the good state, outright trading becomes an alternative to the repo, and expected round-trip costs may impose a bound on implicit opportunity rates (cf. Section III).

$\underline{p}_b > 0$ and $\underline{p}_b$ is a mass point of $\widetilde{p}_b$, or if $\overline{p}_a < \infty$ and $\overline{p}_a$ is a mass point of $\widetilde{p}_a$. In practice, insurance could appear in the form of a guarantee by a third party that is of indisputable standing. Note that the liquidation value of an uninsured collateral portfolio may (but need not) drop to zero with positive probability. Note also that the replacement costs of uninsured collateral may be bounded from above. Finally, we will say that collateral is of *junk quality* if there is some composition y such that the probability that $\widetilde{p}_b$ assumes a value in the open interval $(0; \overline{p}_a)$ is zero. In particular, an asset with a certain short-term liquidation value of $\widetilde{p}_b \equiv 0$ would make a collateral of junk quality.

Theorem 1 (Risk sharing). *Fix i_L, i_B, and $\lambda > 0$. Assume that Assumptions 1 through 4 hold, and that collateral is not insured. Consider any efficient true SRA (y, h, r). Then, provided collateral is imperfect, one finds* $\mathrm{pr}\{\widetilde{v}_b < 1 + r\} > 0$*, where* $\mathrm{pr}\{.\}$ *denotes the unconditional probability. Furthermore, provided that borrower's collateral is neither of junk quality nor exploited, one finds* $\mathrm{pr}\{\widetilde{v}_a > 1 + r\} > 0$.

Thus, under two-sided credit risk, it will typically be efficient to expose both lender and borrower in a standard repurchase agreement to counterparty risk. To see why this is true, assume that the lender is fully protected against any losses from the repo transaction. Then a marginal decrease of the haircut applied on a collateral portfolio may introduce the risk of a small loss for the lender, but this loss occurs, if the collateral is uninsured, only with a small probability. As a consequence, the expected utility of a fully protected lender is not really lowered by a marginal concession in the haircut. However, for the borrower who is not fully protected, a marginal decrease in the haircut reduces losses that occur with strictly positive probability. Therefore, when the lender is fully protected, the lender's reservation price (in terms of the repo rate) for a small concession in the haircut is nil, while the borrower's willingness to pay is strictly positive. Hence, full protection of the lender cannot be efficient. A similar argument shows that full protection of the borrower cannot be efficient under a different set of conditions. Indeed, when the borrower is fully protected, the lender would marginally benefit from a compensated increase in the haircut, at least if collateral is not of junk quality, while the borrower would be indifferent at the margin. Moreover, such a compensated increase in the haircut is feasible provided that the borrower's collateral is not fully exploited. Combining both insights, we find that optimal risk sharing using non-junk, yet imperfect and uninsured collateral either exploits the borrower's collateral or results in true sharing of risk.

II. Optimal collateral

A perspective that is sometimes taken is that there is a conflict of interests between lender and borrower insofar that the lender is interested to obtain the best collateral from the borrower, while the borrower is interested to forward only the worst collateral. As we will see in the present section, this perspective is not completely accurate because it neglects that counterparties negotiate, together with the composition of collateral, also about haircut and interest rate. More specifically, we show now that Theorem 1 has testable implications for the use of collateral in repurchase agreements provided that collateral assets can be ordered along the liquidity dimension. To our knowledge, this is the first result of this type in the literature. We start with an example.

Example 1. Two risk-averse counterparties negotiate over the terms of a standard repurchase agreement. There are two assets that can be used as collateral, i.e., $m = 2$. Asset 1 has an expected conditional liquidation value of $E[\widetilde{p}_b^1] = 0.98$, and an expected conditional replacement cost of $E[\widetilde{p}_a^1] = 1.02$. Asset 2 has an expected conditional liquidation value of $E[\widetilde{p}_b^2] = 0.97$, and an expected conditional replacement cost of $E[\widetilde{p}_a^2] = 1.05$. We wish to formalize the notion that asset 2 is more risky and less liquid than asset 1. To this end, we shall relate "normalized" asset prices. Specifically, it will be assumed that there exists a price mark μ_1 for asset 1 and a price mark μ_2 for asset 2 such that

$$\frac{\widetilde{p}_b^2}{\mu_2} \equiv \frac{\widetilde{p}_b^1}{\mu_1} - \widetilde{\varepsilon}_b^1 \quad \text{and} \quad \frac{\widetilde{p}_a^2}{\mu_2} \equiv \frac{\widetilde{p}_a^1}{\mu_1} + \widetilde{\varepsilon}_a^1 \tag{4}$$

for independent random variables $\widetilde{\varepsilon}_b^1$ and $\widetilde{\varepsilon}_a^1$, where $\equiv$ denotes equality in distribution. The counterparties are now envisaged to consider a collateral composition $(y_1, y_2) = (80\%, 20\%)$, combined with a haircut of $h = 4\%$, and a repo rate of 2%, say. It is straightforward to check that expected conditional prices of the collateral portfolio are given by

$$E[\widetilde{v}_b] = (1 + 4\%)(80\% \cdot 0.98 + 20\% \cdot 0.97) = 1.01712, \tag{5}$$

$$E[\widetilde{v}_a] = (1 + 4\%)(80\% \cdot 1.02 + 20\% \cdot 1.05) = 1.06704. \tag{6}$$

It can now be shown that, provided that the borrower has unused quantities of collateral 1, there is scope for a Pareto improvement. As an illustration, consider the adjusted collateral composition $(y_1', y_2') = (100\%, 0\%)$. In this situation, there are several combinations of haircut and repo rate that achieve a Pareto improvement. For instance, with price marks chosen to be $\mu_1 = 1.00$ and $\mu_2 = 1.01$, counterparties might want to consider a haircut of $h = 4.208\%$ and an unchanged repo rate.[20] Conditional prices of the adjusted collateral portfolio are then given by

$$E[\widetilde{v}_b'] = (1 + 4.208\%)(100\% \cdot 0.98 + 0\% \cdot 0.97) = 1.0212384, \quad (7)$$

$$E[\widetilde{v}_a'] = (1 + 4.208\%)(100\% \cdot 1.02 + 0\% \cdot 1.05) = 1.0629216. \quad (8)$$

Through the adjustment, the expected conditional liquidation value of the collateral portfolio has increased and the expected conditional replacement cost has declined, which is individually beneficial for both counterparties. Moreover, as asset 2 is more risky and less liquid than asset 1, the adjustment strictly reduces the volatility of conditional prices. Thus, the improvement of the collateral allows an improved sharing of the risks resulting from the transaction.

Example 1 illustrates the possibility that, provided that rational counterparties reach an efficient outcome, and credit risk is two-sided, good collateral is used up first in the interbank transaction. The relatively illiquid and risky collateral is not used in the example because it would not allow counterparties to share their risks resulting from the agreement as efficiently as more liquid and less risky collateral. Example 1 thereby describes an effect that might offer an explanation for the empirical regularity discussed in the Introduction that interbank repos rely predominantly on liquid collateral.[21]

Note that if credit risk is one-sided only (and collateral is ample), the economic characteristics of the collateral portfolio should play a subordinated role. In fact, this point is well-known (cf., e.g., Plaut, 1985). In our model, for instance, if the lender cannot default then the borrower could in many cases offer even very risky and illiquid assets as collateral. Indeed, provided that the liquidation value of the

[20] This haircut has been constructed as in the proof of Theorem 2, for $\delta = 0.1996$. Note that the haircut is increasing here because collateral 2 has a higher expected appreciation than collateral 1.

[21] Also security-driven repurchase agreements tend to concentrate on liquid assets. This is because of dynamic shorting strategies that depend on the trader's ability to close the position potentially at very short notice.

collateral asset is bounded away from zero, a sufficiently large haircut would fully protect the lender against any credit risk. If, however, the liquidation value is not bounded away from zero, the borrower of cash would of course offer all available collateral. Conversely, when the borrower cannot default, no collateral is needed in the first place.

The rest of this section merely extends Example 1 into a general statement. We shall impose a linear ranking on the collateral assets.

Assumption 5. (Liquidity ranking) *Fix* $\widetilde{p}_b^0 \equiv \widetilde{p}_a^0 \equiv 1$ *and* $\mu_0 = 1$. *Then, for* $j = 1, ..., m,$

$$\frac{\widetilde{p}_b^j}{\mu_j} \equiv \frac{\widetilde{p}_b^{j-1}}{\mu_{j-1}} - \widetilde{\varepsilon}_b^j \text{ and } \frac{\widetilde{p}_a^j}{\mu_j} \equiv \frac{\widetilde{p}_a^{j-1}}{\mu_{j-1}} + \widetilde{\varepsilon}_a^j, \tag{9}$$

where $\mu_1, ..., \mu_m > 0$ *are constants, and* $\{\widetilde{\varepsilon}_b^1, ..., \widetilde{\varepsilon}_b^m\}, \{\widetilde{\varepsilon}_a^1, ..., \widetilde{\varepsilon}_a^m\}$ *are collections of independent random variables satisfying* $E[\widetilde{\varepsilon}_b^j] > 0, E[\widetilde{\varepsilon}_a^j] > 0$ *for* $j = 1, ...m.$

Just as in Example 1, the random variables $\widetilde{\varepsilon}_b^j$ and $\widetilde{\varepsilon}_a^j$ capture the superior quality of asset $j-1$ compared to asset j (with asset 0 being cash). The liquidity risk of asset j, compared to asset $j-1$, is measured by the volatilities of $\widetilde{\varepsilon}_a^j$ and $\widetilde{\varepsilon}_b^j$. Expected illiquidity of asset j, compared to asset $j-1$, is measured by $E[\widetilde{\varepsilon}_a^j]$ and $E[\widetilde{\varepsilon}_b^j]$. Assumption 5 requires that the ordering is weak in the former of these two dimensions, and strict in the latter.

With these preparations, the following result is obtained.

Theorem 2 (Gresham's law for collateral, market version). *Fix* i_L, i_B, *and* $\lambda > 0$. *Assume that collateral is uninsured. Then, under Assumptions 1 through 5, any efficient true SRA* (y^*, h^*, r^*) *entails the collateral composition*

$$y^* = (\frac{q_1^{i_B}}{1+h^*}, ..., \frac{q_{j^*-1}^{i_B}}{1+h^*}, 1 - \frac{\sum_{j=1}^{j^*-1} q_j^{i_B}}{1+h^*}, \underbrace{0, ..., 0}_{m-j^* \ times}), \tag{10}$$

where j^* *is the smallest index such that* $\sum_{j=1}^{j^*} q_j^{i_B} \geq 1 + h^*$.

Thus, when uninsured assets can be linearly ordered in the liquidity and riskiness dimension, then it is of mutual interest of borrower and lender to use up the most liquid and least risky collateral first.

III. Feasibility of the secured market transaction

While the money market is most of the time a reliable source of funding for many players, it has been known that the unsecured market is prone to break down under stress (cf. Flannery, 1995). In contrast, at least before the liquidity turmoil, it was typically understood that collateral guarantees access to money markets also when credit risk is non-negligible (see, e.g., Allen et al., 1989). In the present section, it is shown that interbank lending may not be feasible even if collateral causes no opportunity costs, information is symmetrically distributed, and physical transaction costs are zero. Sufficient conditions for a market imperfection are that both banks default with positive probability and that assets that are available as collateral are either not perfectly liquid or not absolutely risk-free.

Indeed, counterparties will approve a contract only when it is individually rational to do so. In practice, effective outside options might include capital market transactions (a private bond placement, say), outright transactions such as a straight debt sale (provided that collateral assets are marketable at the time of contracting, cf. footnote 19), money market transactions with non-banks, recourses to the central bank's standing facilities, renegotiation of contractual obligations, accepting a contractual penalty, etc. In the worst case, banks might even become more reluctant to offer credit to non-banks. We will assume here outside options guaranteeing utility levels of $\underline{u}_L = (\pi_G + \pi_B)u_L(r^D)$ to the lender and $\underline{u}_B = (\pi_G + \pi_L)u_B(-r^B)$ to the borrower, respectively, where $r^D = r^D(i_L)$ is the lender's implicit risk-free opportunity deposit rate, and $r^B = r^B(i_B)$ is the borrower's implicit unsecured opportunity borrowing rate.

As an illustration, consider a fixed collateral composition. For any given opportunity rate r^D, denote by $\rho^D(h) \geq r^D$ the lowest repo rate that a lender would be willing to accept for a given haircut h. Similarly, for any given r^B, denote by $\rho^B(h) \leq r^B$ the highest repo rate that the borrower would accept for a given haircut h. Figure 2 illustrates $\rho^D(h)$ and $\rho^B(h)$ for a numerical example. Both cut-off rates are declining in the haircut because a higher haircut implies improved (weakened) protection for the lender (borrower) that must be compensated by a lower (lower) repo rate. Clearly, consistent with Theorem 2, the use of more liquid and less risky collateral will lower $\rho^D(h)$, increase $\rho^B(h)$, and thereby make the market transaction

Figure 2 about here

(weakly) more likely. A market imperfection results if even for the best collateral composition, there is no combination of repo rate and haircut that lies both above $\rho^D(.)$ and below $\rho^B(.)$.

Example 2. For another illustration, assume that both lender and borrower are risk-neutral. Assume also that the liquidation value and the replacement cost of the only available collateral asset is known to be P_b and P_a with certainty at date 1, respectively, where $P_a > P_b > 0$. Consider first the lender. Expected utility at the time of contracting is given by

$$E[\widetilde{u}_L] = \pi_G r + \pi_B \min\{(1+h)P_b - 1; r\}. \tag{11}$$

It is not difficult to see that in any contractible (i.e., efficient) agreement (y, h, r), the lender will not be overprotected, i.e.,

$$(1+h)P_b \leq 1 + r. \tag{12}$$

This is because overprotection would be without value for the lender, but costly for the borrower. Thus,

$$E[\widetilde{u}_L] = \pi_G r + \pi_B((1+h)P_b - 1). \tag{13}$$

Comparing these expressions with the available outside option $\underline{u}_L$ for the lender yields that for a deposit rate r satisfying condition (12) of at least

$$\rho^D(h) = r^D + \frac{\pi_B}{\pi_G}(1 + r^D - (1+h)P_b), \tag{14}$$

the lender would be willing to contract against a haircut of h. On the other hand, when (12) is not satisfied, then the lender would be overprotected, and expect at least r^D. Thus, in general,

$$\rho^D(h) = r^D + \frac{\pi_B}{\pi_G}(1 + r^D - (1+h)P_b)^+, \tag{15}$$

where, as usual, $(x)^+ = x$ for $x > 0$ and $= 0$ otherwise. Using completely analogous arguments, one can see that the borrower would be willing to contract against a haircut of h if and only if the repo rate is at most

$$\rho^B(h) = r^B + \frac{\pi_L}{\pi_G}(1 + r^B - (1+h)P_a)^-, \tag{16}$$

where $(x)^- = x$ for $x < 0$ and $= 0$ otherwise.

Apparently, a repurchase agreement (r, h) is contractible between borrower and lender if and only if $\rho^D(h) \leq \rho^B(h)$ for some h. As the expressions (15) and (16) are piecewise linear, one can check that a contract is not feasible if and only if conditions

$$\rho^D(\frac{1+r^B}{P_a} - 1) > r^B \tag{17}$$

and

$$\rho^B(\frac{1+r^D}{P_b} - 1) < r^D \tag{18}$$

are simultaneously satisfied. Rewriting (17) and (18) yields

$$\frac{\pi_B}{\pi_G + \pi_B} \cdot \frac{P_a - P_b}{P_a} > \frac{r^B - r^D}{1 + r^B} \tag{19}$$

and

$$\frac{\pi_L}{\pi_G + \pi_L} \cdot \frac{P_a - P_b}{P_b} > \frac{r^B - r^D}{1 + r^D} \tag{20}$$

as intuitive conditions for contractibility. That is, in the case of risk-neutrality and risk-free but illiquid collateral, contracting is impossible if and only if both (19) and (20) are satisfied.[22]

The following result shows that the illustrated possibility is not due to the simplifying assumptions of the previous example.

Theorem 3. **(Market imperfection)** *Fix i_L, i_B, and $\lambda > 0$. Let Assumptions 1 through 4 be satisfied. Assume that collateral is imperfect. Then, for any interest rate level $r_0 > 0$ and for any collateral composition y, there is an implicit unsecured borrowing rate r^B for the borrower and an implicit risk-free deposit rate r^D for the lender such that $r^B > r_0 > r^D$, and such that with these opportunity rates, no market transaction is individually rational for both lender and borrower.*

The analysis thereby also suggests that during times of financial distress and mutual distrust, counterparties may be *generally* unwilling to exchange liquidity, even against collateral. Under stress, several mutually reinforcing developments are likely to amplify the basic effect. First, banks may perceive a higher probability of an individual default. Second, perceptions of potential illiquidity and riskiness of collateral

[22] A closer inspection of Example 2 also shows that with a degenerate price distribution, it can indeed be efficient to protect one counterparty fully against credit risk.

may increase, making it more difficult to find conditions that are mutually satisfactory. Third, counterparties may also become more risk-averse. Fourth, there may be the fear that needs of liquidity increase further. Finally, even if a counterparty would be willing to give cash for collateral today, this counterparty may not be confident that the collateral will be accepted tomorrow. The joint effect of such developments might lead to a disruption even of the "secured" segment of the interbank market.[23]

Theorem 3 might relate to three distinct developments during the recent liquidity crunch. First, a market failure might have been a motivation for the ECB and the Swiss National Bank to offer U.S. Dollar funds to euro-area and Swiss counterparties since December 2007 through a participation in the U.S. Fed's term auction facility (TAF). To understand why, note that a euro-area counterparty in search of dollar funding would in principle have been able to access euro funding through the Eurosystem's open market operations. Apparently, however, there has been a problem with turning this euro funding into dollars, which normally could be done by a forex swap transaction with some U.S. bank. While such swaps differ from repos in many aspects, the underlying economic structure is similar when one currency is interpreted as the collateral security. Our theory would suggest that if there is two-sided counterparty risk, and if exchange rates are volatile, it may be difficult for euro-area counterparties to obtain dollar funding.

Another visible market disruption that could be mentioned here is the repo run on dealers in the U.S. in March 2008, related to the near-fall of Bear Stearns. The Bank of England (2008, p. 9) writes that "Bear Stearns was not only unable to obtain funding in unsecured markets, but also could not secure funds against high-quality collateral. That led to a rapid fall in its sizable reserves of liquid assets ... and the firm was forced to seek support from JPMorgan Chase & Co. and the Federal Reserve Bank of New York." Apparently also other primary dealers had difficulties in obtaining short-term funding. In a quite unconventional move, the Federal Reserve decided, effective on Tuesday, March 11, 2008, to offer primary dealers an amount of $200 bn in Treasury bonds and bills in exchange for mortgage-backed securities.

[23] This disruption should be especially strong when the sole collateral available is positively correlated with the borrower's equity. Conversely, the analysis suggests that a borrower may find it easier to transact in the interbank market by offering the lender collateral whose market value is positively correlated with the *lender*'s equity, such as the lender's own uncovered bonds.

The new element of that so-called Term Securities Lending Facility (TSLF) has been that individual transactions have a term of 28 days (rather than overnight).[24] The Fed went further by implementing, effective March 16, the so-called Primary Dealer Credit Facility (PDCF) that offers penalty-free access to overnight repo loans against a range of collateral assets that strictly includes securities accepted in open market operations. In contrast to the discount window, this facility is open to primary dealers of the Federal Reserve Bank of New York, yet not to depository institutions. In the specific case of Bear Stearns, funding problems might have resulted either from insufficiency of collateral (which would preclude lending even in the case of one-sided credit risk), fears by potential lenders of getting involved in complicated default procedures, and even outright predatory behavior. Still, Theorem 3 captures an effect that generally might have increased frictions in the repo market, and which could have contributed indirectly to the debacle.

Finally, it will be recalled that on August 9, 2007, problems with subprime loans in the U.S. led, among other things, to a sudden dry-out of the market for asset-backed commercial paper (ABCP), which had served as a source of funding for so-called structured investment vehicles (SIVs). Banks with credit commitments vis-à-vis such vehicles had an unexpected increase in liquidity needs. Illiquid assets held by the vehicles, such as collateralized debt obligations (CDOs), could no longer serve as collateral. At the same time, those investors that had refused to roll over commercial paper have received significant cash transfers to their bank accounts. Kashyap et al. (2002) have put forward the argument that commercial banks have the unique ability to pool imperfectly correlated liquidity risks resulting from loan commitments and deposit contracts. Gatev and Strahan (2006) find empirical support for a similar mechanism in the context of the commercial paper market. The stylized facts mentioned above might relate our analysis to the pooling argument. Specifically, one could argue that before the turbulences, numerous banks might have decided to specialize and to exploit the synergies identified by Kashyap et al. across the money market, assuming that liquidity risks can be shared effectively with other banks. Then, during the turbulence, some of those banks (e.g., invest-

[24]More recently, the Bank of England has implemented a similar measure, the so-called Special Liquidity Scheme, which offers terms of one year, renewable to up to three years.

ment banks) would have to satisfy a loan commitment, while others would receive a liquidity inflow in the form of additional deposits. However, in view of Theorem 3, a market transaction that matches supply and demand may not be guaranteed. Thus, using the terminology introduced by Kashyap et al., with specialized banks, synergies across banks may become a prerequisite to synergies across the two sides of the balance sheet.

Under normal market conditions, the analysis suggests that also in the secured segment of the money market, a counterparty may be constrained to trading with a counterparty that has a relatively good credit standing. Under normal market conditions, this effect should be reflected in the topology of the interbank network. Two types of regularities are predicted. First, counterparties with an excellent rating may be able to intermediate in the repo market. In practice, this should lead to a *two-tiered structure* of the repo market, just as predicted for the unsecured market by Freixas and Holthausen (2004). The second regularity should be the emergence of *central counterparty* trading, where a clearing house with good standing intermediates the transaction by becoming a counterparty to both the lender and the borrower.

IV. Central bank collateral and haircuts

In Section II, it has been shown that with two-sided credit risk, counterparties seek to use the most liquid and least risky assets as collateral first. What is the impact of this effect on central bank collateral? And how can haircuts be used by central banks to steer the composition of central bank collateral? To address these issues, an extension of the basic model will be considered in which banks forward collateral also to the central bank. Examined will be stable compositions of central bank collateral, and the role of haircuts.

Thus, in contrast to the set-up considered so far, there is now a central bank, and it is assumed that Bank 1 and Bank 2 have debt of $D_1 \geq 0$ and $D_2 \geq 0$, respectively, outstanding vis-à-vis the central bank from date 0 onwards (cf. also Figure 1). Moreover, the size of the liquidity shock λ is now the realization of a random variable $\widetilde{\lambda} > 0$ with support $\mathbb{R}_{\geq 0}$. Under Assumptions 2 through 4, expected utilities at the time of contracting are given by

$$E[\widetilde{u}_L] = \pi_G u_L(\lambda r) + \pi_B E[u_L(\lambda \min\{\widetilde{v}_b - 1; r\})], \tag{21}$$

and

$$E[\widetilde{u}_B] = \pi_G u_B(-\lambda r) + \pi_L E[u_B(\lambda \min\{1 - \widetilde{v}_a; -r\})]. \tag{22}$$

The central bank exerts its influence on the money market by two policy choices. At date 0, a decision is made on the central bank's *collateral policy*. Specifically, it is assumed that the central bank chooses the least liquid eligible asset m_{CB}, where $1 \leq m_{\mathrm{CB}} \leq m$. Thus, only assets contained in the set $J = \{1, ..., m_{\mathrm{CB}}\}$ will be accepted as collateral in central bank operations. Let $\eta_j \geq 0$ denote the exogenous haircut applied by the central bank to asset $j \in J$. Note that in contrast to the interbank market, these haircuts are not subject to negotiation. Obviously, collateral policy and haircuts must be consistent with what is available in the market, i.e.,

$$D_i \leq \sum_{j=1}^{m_{\mathrm{CB}}} \frac{q_j^i}{1+\eta_j} \qquad (i = 1, 2). \tag{23}$$

Second, the central bank exerts influence on the money market by affecting the relative bargaining power of lender and borrower in the market. For specificity, it is assumed that the central bank chooses, immediately following the liquidity shock, a *liquidity policy* $\alpha = (\alpha_L, \alpha_B)$ such that $\alpha_L \geq 0$, $\alpha_B \geq 0$, and $\alpha_L + \alpha_B = 1$; counterparties then determine the terms of the SRA at the contracting stage using the Nash bargaining solution, where α_L and α_B represent the parameters measuring bargaining power of the lender and borrower, respectively.

Denote by $\theta^i = (\theta_1^i, ..., \theta_m^i)$ the composition of bank i's collateral deposits, net of haircuts, with the central bank at date 0. Note that by definition of the collateral policy, $\theta_j^i = 0$ for $j \notin J$. Moreover, θ^i must be *feasible*, i.e.,

$$(1+\eta_j)\theta_j^i D_i \leq q_j^i \qquad (j = 1, ..., m). \tag{24}$$

In line with the institutional environment in the euro area, it is assumed that each bank i may at any point in time change the feasible collateral composition with the central bank as long as the total market value of the collateral net of haircuts remains at least D_i. Such replacement may indeed occur, in particular when bank i_B

that turns out to be the borrower wishes to redeploy liquid assets to free collateral for an interbank transaction.

Substitution is not necessary, though. In our framework, there are in principle two reasons why relatively liquid collateral may be kept with the central bank. One potential reason is that the maximal size of the liquidity shock expected in the interbank market is small, so that there is no need to optimize collateral that had been placed with the central bank. The reader will note that we have chosen to exclude this possibility by imposing a full-support assumption on $\widetilde{\lambda}$, but it is clear that dropping this assumption may yield (partial) indeterminacy of central bank collateral. The second potential reason for not optimizing collateral held with the central bank is that a secured market transaction is correctly expected not to come about. To exclude this possibility, we impose another assumption. Specifically, we will assume that a repo market imperfection may occur only when the liquidity shock is excessively large, i.e., only when the shock exceeds the repurchase price of the borrower's non-cash assets in a worst-case scenario.

Assumption 6. (No crowding-out) *For $i_B = 1, 2$, there is no market imperfection for any $\lambda < \sum_{j=1}^{m_{\mathrm{CB}}} q_j^i \overline{p}_a^j$, where $\overline{p}_a^j$ denotes the supremum of the support of $\widetilde{p}_a^j$ for $j \in J$.*

The following definition turns out to be useful. For a given collateral policy J, a pair of collateral compositions (θ^1, θ^2) for Banks 1 and 2, respectively, will be called *stable* if there is, for any realization of $i_B = 1, 2$, and for any realized liquidity shock $\lambda > 0$, either a market imperfection or an efficient true SRA between Banks 1 and 2 that does not imply the replacement of collateral deposited with the central bank. We are ready to formally describe the residual nature of central bank collateral.

Theorem 4. (Gresham's law for collateral, central bank version) *Assume that collateral is uninsured. Then, under Assumptions 1 through 6, the unique stable pair of collateral compositions $(\theta^1(J), \theta^2(J))$ is given by*

$$\begin{aligned}\theta^i(J) \quad = \quad & (\underbrace{0, ..., 0}_{j_*(i)-1 \text{ times}}, 1 - \sum_{j=j_*(i)+1}^{m_{\mathrm{CB}}} \frac{q_j^i}{(1+\eta_j)D_i}, \frac{q_{j_*(i)+1}^i}{(1+\eta_{j_*(i)+1})D_i}, \cdots \\ & ..., \frac{q_{m_{\mathrm{CB}}}^i}{(1+\eta_{m_{\mathrm{CB}}})D_i}, \underbrace{0, ..., 0}_{m-m_{\mathrm{CB}} \text{ times}}), \qquad (25)\end{aligned}$$

where $j_*(i)$ denotes the largest index such that $\sum_{j=j_*(i)}^{m_{\mathrm{CB}}} q_j^i/(1+\eta_j) \geq D_i$, and $i = 1, 2$.

Theorem 4 captures the observations discussed in the Introduction by suggesting that commercial banks have an incentive to use less liquid and more risky assets with preference in central bank operations. Indeed, as more liquid and less risky assets allow a better risk sharing in interbank repo transactions, there is an endogenous opportunity cost of taking the most liquid and least risky assets to the central bank. Moreover, the residual nature of central bank collateral should become more evident in times of increasing liquidity risks.

Our analysis might also help to clarify the role of haircuts applied by the central bank. Haircuts have always been an instrument of risk management, both for commercial banks and for central banks. However, as Theorem 4 suggests, there is only a very limited role for haircuts as an instrument to steer the composition of central bank collateral. Indeed, the opportunity costs of using the least liquid and most risky assets accepted by the central bank will remain negligible as long as the borrower's holdings of such assets are ample enough. Changing haircuts should therefore not be sufficient to induce commercial banks to take more liquid and less risky collateral to the central bank. In particular, our theory suggests that haircuts are not an instrument for fine-tuning the composition of *liquid* collateral along, say, issuing fiscal authorities. This point addresses a question of significant practical interest (see Buiter and Sibert, 2005, and references given therein).[25]

V. Welfare implications

To the extent that the proper working of funding markets depends on the availability of sufficient amounts of liquid collateral, a policy issue may arise when central bank operations have the potential to withhold such collateral from uses in the interbank market. To explore this issue, we will use now the extension introduced in the previous section to examine the consequences on welfare of changing the central bank's collateral policy.[26]

To evaluate the welfare consequences of the collateral framework, it is useful to

[25] Alternatively, one might want to apply different pricing to different collateral, e.g., by using variable-rate tenders for given quantities in each liquidity basket. However, this strategy may not be practicable under all circumstances (cf. Federal Reserve Bank of New York, 2008).

[26] Imperfections of collateral assets interact with policy objectives also in Kocherlakota (2001), where risky collateral is used to rationalize deposit insurance. The question of why apparently all central banks do require collateral is not addressed in the present paper. For a comprehensive discussion of this point, see ECB (2007b).

note that the central bank is always in the position to effectively limit its exposure from repo operations involving counterparties that have ample collateral. Indeed, given its standing as a monetary authority, our earlier remark at the end of Section I should apply also here, i.e., there is no market disruption even when haircuts required to limit the central bank's exposure are relatively large. Motivated by this consideration, we will analyze welfare without explicit reference to the central bank and exclusively in terms of expected utilities for lender and borrower.

Two hypothetical scenarios are compared now where the central bank may either pursue a tight or a generous stance concerning the acceptance of collateral. Moreover, adding realism, we will allow that the borrower's opportunity rate $r^B = r^B(i_B, J)$ may depend also on the central bank's collateral framework. Denote by $E[\widetilde{u}_B|(J,\alpha)]$ and $E[\widetilde{u}_L|(J,\alpha)]$, respectively, expected utility of borrower and lender at the time of contracting, given the central bank adheres to policy choices (J,α), and given that counterparties' collateral compositions with the central bank are stable with respect to J.

Theorem 5. (Welfare consequences) *Let Assumptions 1 through 5 be satisfied. Fix some policy (J,α), a collateral set $J' = \{1,...,m'_{\mathrm{CB}}\} \supseteq J$, a liquidity shock $\lambda > 0$, and some i_B. Assume that $r^B(i_B,J') \leq r^B(i_B,J)$. Then for $E[\widetilde{u}_B|(J,\alpha)] \geq \underline{u}_B(J')$, there is a liquidity policy α' such that $E[\widetilde{u}_B|(J',\alpha')] \geq E[\widetilde{u}_B|(J,\alpha)]$ and $E[\widetilde{u}_L|(J',\alpha')] \geq E[\widetilde{u}_L|(J,\alpha)]$.*

Theorem 5 contains a prediction concerning the welfare implications of an expanded collateral set. It says that if an extension of the set of collateral assets accepted by the central bank is accompanied by an appropriate liquidity policy α', expected utilities for both lender and borrower may increase. The reason is that a less restrictive collateral policy allows counterparties to use more liquid and less risky collateral in the interbank repo market.

Maybe it should be stressed at this point that the weak increase of expected utility for both lender and borrower implies that the certainty-equivalent interest rates for the two counterparties move closer together. In fact, by definition, any liquidity policy α' that, compared to the tight collateral regime J combined with liquidity policy α, increases the implicit deposit rate and decreases the implicit borrowing rate, will produce the welfare gain. The policy change suggested by

Theorem 5 is therefore consistent with the view that the central bank is mainly in the market to steer interbank conditions, and that welfare maximization through the collateral framework is subject to this important constraint.[27]

To illustrate Theorem 5, we mention the cases of the U.S. Fed, the Bank of England, the Bank of Canada, and the Bank of Australia. Before the start of the turmoil in August 2007, these central banks generally accepted only a very narrow range of assets, mainly government bonds, as collateral. During the turbulences, however, all of these institutions significantly broadened the range of eligible collateral. Theorem 5 provides a rationale for such policy adjustment.

A couple of caveats apply, however. First, for the case $E[\widetilde{u}_B|(J,\alpha)] < \underline{u}_B(J')$ that is not considered in Theorem 5, we note that the lender might in principle be worse off if the expansion of the collateral set improves the outside option for the borrower. There are two potential reasons for this. One reason is that the lender might have had a very strong bargaining position in the tight environment, which is lost when the central bank changes its policy. Another potential reason is that there may be a crowding-out of the market transaction. This, too, may mean a loss for the lender, but again only when her bargaining position under the tight policy had been strong. On the other hand, the loss of interim utility for the lender may sometimes be more than compensated from an ex ante perspective when the roles of lender and borrower are not yet assigned.

Moreover, in view of Theorems 2 and 3, it may well be that the collateral potentially unleashed by an enlargement of the set of eligible collateral will not be used in the market. It could be argued that this is the present situation in the euro area given that the Eurosystem already accepts a very broad list of assets as collateral. Then, it would not be the case that too much precious collateral is bound in transactions with the central bank. Widening the set of eligible collateral would, therefore, be unlikely to re-establish the proper working of the money market. Indeed, the current problems in the euro repo market seem to be linked rather to a general concern

[27] Collateral policy might affect market activity in other ways than suggested by Theorems 4 and 5. Firstly, the usual *moral hazard* caveat applies. After all, accepting illiquid collateral, especially during times of market stress, works like an insurance of commercial banks against temporary funding problems. Secondly, to the extent that repricing risk of illiquid assets may trigger margin calls, *liquidity risks* of commercial banks might actually increase. Finally, there may be an impact on *relative asset prices*.

about the quality of collateral assets and to a mutual mistrust between commercial banks.[28]

VI. Conclusion

Modern liquidity management increasingly relies on repurchase agreements through which cash is exchanged short-term against collateral assets of longer maturities. Interestingly, almost all such refinancing is based on securities that are very stable in value and actively traded. Market requirements on asset liquidity became even stricter when interbank market conditions tightened, as during the credit crunch following August 2007. On the other hand, there has been a tendency to deposit more and more illiquid assets for use in central banks' liquidity-providing operations.

The present study has derived a number of theoretical predictions that clarify and explain these and related observations. First, it has been shown that if there is a choice of collateral in a market transaction, then the most liquid and least risky asset will allow counterparties to achieve the most efficient risk-sharing. However, if the best collateral available is still relatively illiquid or risky, and if there is non-negligible bilateral counterparty risk, then no market transaction may come about at all. This point has allowed us to apply a theoretical argument put forward recently by Kashyap et al. (2002). As regards to policy implications, it has been shown that a less restrictive collateral policy applied by a central bank may lead to a welfare improvement for market participants. Yet, the analysis also suggests that essentially unaffected by the haircut requirement, the least liquid and most risky assets will be deposited with the central bank.

The analysis provides a rationale for the decisions of several central banks to broaden the range of assets accepted as collateral during the turmoil that started in August 2007. For the euro area, the analysis comes to the conclusion that a widening of the set of eligible collateral would not necessarily be (or have been) supportive for a resolution of market disruptions. As there is no evidence that too much high quality collateral is bound in central bank operations, the benefit of unleashing collateral of

[28]To the extent that the *precautionary* demand for collateral that can be used with the Eurosystem is high, as suggested by media reports (cf. Financial Times, 2008), a relaxation of the criteria for collateral would of course help to improve commercial banks' outside option in case of market breakdown. However, the comparative statics of feasibility (cf. Section III) suggests that this would make a market breakdown even more likely.

intermediate liquidity into the market might turn out to be very limited. Instead, problems with secured lending seem to be related to a general concern about the quality of collateral assets and to a mutual mistrust in particular between banks. The situation might have been different in the United States. Since the start of the market turbulences, the Federal Reserve System has repeatedly taken measures that aimed at making a broader collateral base available. Also other central banks have followed this route. As our analysis shows, such measures will be directly beneficial for the banking sector to the extent that illiquidity of collateral assets impairs the functioning of the money market.

There are a number of issues that the present paper could have dealt with, but which as we believe, deserve a separate analysis. To start with, an important omission is that we do not discuss pricing of repo contracts, even though our framework would have allowed us to make definite predictions. In fact, in many cases, it is not difficult to write down, for a given collateral, an equation characterizing the repo rate as a function of default probabilities and the respective conditional distributions of liquidation values and replacement costs. Anecdotal evidence suggests here that repo rates are higher for less liquid assets. We conjecture that this follows from our framework when downside risks are more pronounced than upside risks. Indeed, illiquidity would then be more a problem of the cash lender who must fear that the value of the collateral could drop even more strongly, than for the borrower who will not care so much about potentially increasing replacement costs. A second issue that may be interesting to look at is to endogenize outside options for repo counterparties by embedding the model into a search environment. In that case the outside options would be the expected utility from meeting another potential OTC counterparty reduced by the cost of having to search the next counterparty. Still another point that we did not address in the paper is a departure from the linear liquidity ordering. This point looks subtle. Intuitively, when assets cannot be ordered linearly in the liquidity dimension, what one is looking for is a portfolio composition that is, when sold, a good hedge for the borrower's equity, but when to be replaced, positively correlated in value with the lender's equity. At this stage, we have no solution to this problem. However, market practice suggests that diversification might be of little value in itself even when collateral cannot be linearly ordered in terms on liquidity.

Appendix: Proofs

Proof of Theorem 1. Fix some efficient true SRA (y, h, r). As shown in Section I, under Assumptions 2 through 4, the lender's expected utility at the time of contracting is given by

$$E[\widetilde{u}_L] = \pi_G u_L(r) + \pi_B \int u_L(\min\{(1+h)p_b - 1; r\})dF_b(p_b), \tag{26}$$

where $F_b(p_b) = \text{pr}\{\widetilde{p}_b \leq p_b\}$ denotes the cumulative distribution function of $\widetilde{p}_b$. The integrand in (26) will be $u_L(r)$ for all $p_b > p^* = (1+r)/(1+h)$, and $u_L((1+h)p_b - 1)$ otherwise. Consequently,

$$\begin{aligned} E[\widetilde{u}_L] &= (\pi_G + \pi_B(1 - F_b(p^*)))u_L(r) + \pi_B \int_{p_b \leq p^*} u_L((1+h)p_b - 1)dF_b(p_b) \\ &= (\pi_G + \pi_B)u_L(r) - \pi_B(1+h) \int_{p_b \leq p^*} F_b(p_b)u'_L((1+h)p_b - 1)dp_b, \end{aligned} \tag{27}$$

where we applied integration by parts on the Stieltjes integral. Using Leibniz' rule and $\partial p^*/\partial r = 1/(1+h)$, one obtains

$$\frac{\partial E[\widetilde{u}_L]}{\partial r} = (\pi_G + \pi_B(1 - F_b(p^*)))u'_L(r). \tag{28}$$

A similar calculation starting from (27) and involving $\partial p^*/\partial h = -p^*/(1+h)$ yields

$$\begin{aligned} \frac{\partial E[\widetilde{u}_L]}{\partial h} &= -\pi_B \int_{p_b \leq p^*} F_b(p_b)u'_L((1+h)p_b - 1)dp_b \qquad (29) \\ &\quad + \pi_B p^* F_b(p^*)u'_L(r) - \pi_B(1+h) \int_{p_b \leq p^*} F_b(p_b)p_b u''_L((1+h)p_b - 1))dp_b \\ &= \pi_B p^* F_b(p^*)u'_L(r) - \pi_B \int_{p_b \leq p^*} F_b(p_b)d(p_b u'_L((1+h)p_b - 1)) \qquad (30) \end{aligned}$$

Integrating again by parts,

$$\frac{\partial E[\widetilde{u}_L]}{\partial h} = \pi_B \int_{p_b \leq p^*} p_b u'_L((1+h)p_b - 1)dF_b(p_b). \tag{31}$$

From (28) and (31), the lender's marginal rate of substitution between haircut and repo rate is given by

$$\text{MRS}^L_{h,r} = \frac{\partial E[\widetilde{u}_L]/\partial r}{\partial E[\widetilde{u}_L]/\partial h} = \frac{(\pi_G + \pi_B \text{pr}\{\widetilde{p}_b > p^*\})u'_L(r)}{\pi_B \int_{p_b \leq p^*} p_b u'_L((1+h)p_b - 1)dF_b(p_b)}. \tag{32}$$

A completely analogous derivation yields the borrower's marginal rate of substitution

$$\mathrm{MRS}^B_{h,r} = \frac{\partial E[\widetilde{u}_B]/\partial r}{\partial E[\widetilde{u}_B]/\partial h} = \frac{(\pi_G + \pi_L \mathrm{pr}\{\widetilde{p}_a < p^*\})u'_B(-r)}{\pi_L \int_{p_a \geq p^*} p_a u'_B(1-(1+h)p_a)dF_a(p_a)}, \tag{33}$$

where $F_a(.)$ denotes the distribution function of $\widetilde{p}_a$. To provoke a contradiction, assume that the lender is fully protected under (y, h, r), i.e., assume $\mathrm{pr}\{\widetilde{v}_b < 1+r\} = 0$. Since $h > -1$, this implies $\mathrm{pr}\{\widetilde{p}_b < p^*\} = 0$. Hence, $p^* \leq \underline{p}_b$. Moreover, since $r > -1$, also $p^* > 0$. Thus, given that collateral is not insured, p^* is not a mass point of $\widetilde{p}_b$, so that even $\mathrm{pr}\{\widetilde{p}_b \leq p^*\} = 0$. From Assumption 1, we must have $\pi_G + \pi_B \mathrm{pr}\{\widetilde{p}_b > p^*\} > 0$. Thus $\mathrm{MRS}^L_{h,r} = \infty$. On the other hand, $\mathrm{pr}\{\widetilde{p}_b \geq p^*\} = 1$, and so, as collateral is imperfect, $\mathrm{pr}\{\widetilde{p}_a \leq p^*\} < 1$ or, equivalently, $\mathrm{pr}\{\widetilde{p}_a > p^*\} > 0$. Using (33) and again Assumption 1 yields $\mathrm{MRS}^B_{h,r} < \infty$, which implies that counterparties would jointly prefer to use a lower haircut. Contradiction. Hence, $\mathrm{pr}\{\widetilde{v}_b < 1+r\} > 0$. Assume now that in addition, collateral is not of junk quality, and not all collateral is exploited. If then the borrower were fully protected under (y, h, r), i.e., if $\mathrm{pr}\{\widetilde{v}_a > 1+r\} = 0$, then from $h > -1$, we would have $\mathrm{pr}\{\widetilde{p}_a > p^*\} = 0$. Then, clearly, $p^* \geq \overline{p}_a$. As collateral is not insured, p^* cannot be a mass point of $\widetilde{p}_a$, hence even $\mathrm{pr}\{\widetilde{p}_a \geq p^*\} = 0$ and the denominator in (33) vanishes. Moreover, using Assumption 1, one finds $\mathrm{MRS}^B_{h,r} = \infty$, i.e., the borrower would be willing to accept a small increase in the haircut essentially without any compensation in the repo rate. On the other hand, as collateral is not of junk quality, and because $p^* \geq \overline{p}_a$, we have $\mathrm{pr}\{0 < \widetilde{p}_b \leq p^*\} > 0$. Hence, using Assumption 1 again, the denominator in (32) does not vanish, and so $\mathrm{MRS}^L_{h,r} < \infty$, i.e., the lender would be willing to compensate the borrower for a non-marginal increase in the haircut by a non-marginal reduction in the repo rate. Since not all collateral is exploited, an increase in the haircut is indeed feasible, and a discrepancy in marginal rates of substitution cannot be efficient. Contradiction. Thus, $\mathrm{pr}\{\widetilde{v}_a > 1+r\} > 0$, which proves also the second claim. □

Proof of Theorem 2. Consider an efficient true SRA $C = (y, h, r)$ with collateral composition $y = (y_1, ..., y_m)$. By definition, C is valid. It suffices to show that it is Pareto dominated for lender and borrower to simultaneously use one collateral asset and not fully use up another collateral with a lower index. To provoke a contradiction, assume that $y_{k+1} > 0$ and $(1+h)y_k < q_k^{i_B}$ for some $k \in \{0, ..., m-1\}$.

From Assumption 5, we know that there are price marks $\mu_1 > 0, ..., \mu_m > 0$, as well as collections of independent random variables $\{\widetilde{\varepsilon}_b^1, ..., \widetilde{\varepsilon}_b^m\}, \{\widetilde{\varepsilon}_a^1, ..., \widetilde{\varepsilon}_a^m\}$ with strictly positive means, such that for any $j = 1, ..., m$,

$$\frac{\widetilde{p}_b^j}{\mu_j} \equiv \frac{\widetilde{p}_b^{j-1}}{\mu_{j-1}} - \widetilde{\varepsilon}_b^j \text{ and } \frac{\widetilde{p}_a^j}{\mu_j} \equiv \frac{\widetilde{p}_a^{j-1}}{\mu_{j-1}} + \widetilde{\varepsilon}_a^j, \tag{34}$$

where $\widetilde{p}_b^0 \equiv \widetilde{p}_a^0 \equiv 1$ and $\mu_0 = 1$. To achieve a Pareto improvement, we seek a new SRA (y', h', r') with $y' = (y'_1, ..., y'_m)$ such that notional amounts in each asset class satisfy $y'_k(1 + h') > y_k(1 + h)$, yet also $y'_{k+1}(1 + h') < y_{k+1}(1 + h)$, and finally $y'_j(1+h') = y_j(1+h)$ for all $j \neq k, k+1$. This can be achieved as follows. Let $\delta \geq 0$ be small. Define the new SRA $C'(\delta) = (y', h', r')$ by

$$\begin{aligned} h' &= \frac{1+h}{1 - (\mu_{k+1}/\mu_k - 1)\delta} - 1, & (35)\\ y'_k &= (1 - (\mu_{k+1}/\mu_k - 1)\delta)y_k + \delta\mu_{k+1}/\mu_k, & (36)\\ y'_{k+1} &= (1 - (\mu_{k+1}/\mu_k - 1)\delta)y_{k+1} - \delta, & (37)\\ y'_j &= (1 - (\mu_{k+1}/\mu_k - 1)\delta)y_j \qquad (j \neq k, k+1), & (38) \end{aligned}$$

and $r' = r$. Clearly, $C'(0) = C$, and for $\delta > 0$ small enough, the haircut h' is well-defined. Moreover, using (3), (35), $y_{k+1} > 0$, and $(1 + h)y_k < q_k^{i_B}$, it is straightforward to check that for δ small enough, we have $0 \leq (1 + h')y'_j \leq q_j^{i_B}$ for $j = 1, ..., m$. Another straightforward calculation exploiting (36) through (38) as well as $\displaystyle\sum_{j=1}^m y_j = 1$ shows that $\displaystyle\sum_{j=1}^m y'_j = 1$. Hence, for $\delta > 0$ small enough, the contract $C'(\delta)$ is well-defined and valid. It is claimed now that for $\delta > 0$ small enough, the SRA $C'(\delta)$ achieves a strict Pareto improvement over C. To see why, consider the conditional liquidation value $\widetilde{v}'_b = (1 + h')\sum_{j=1}^m \widetilde{p}_b^j y'_j$ of the collateral portfolio deposited under the new agreement. Using (36) through (38), one obtains

$$\widetilde{v}'_b = (1+h)\sum_{j=1}^m \widetilde{p}_b^j y_j + (1+h')\frac{\mu_{k+1}}{\mu_k}\delta\widetilde{p}_b^k - (1+h')\delta\widetilde{p}_b^{k+1}. \tag{39}$$

Recall that $\widetilde{v}_b = (1 + h)\widetilde{p}_b$. Then, using (34) for $j = k + 1$ delivers

$$\widetilde{v}'_b \equiv \widetilde{v}_b + (1+h')\delta\mu_{k+1}\widetilde{\varepsilon}_b^{k+1}. \tag{40}$$

An induction argument involving Assumption 5 shows that

$$\widetilde{v}_b \equiv (1+h)\sum_{j=1}^m y_j\widetilde{p}_b^j \equiv (1+h)\sum_{j=1}^m y_j\mu_j - \sum_{j=1}^m \gamma_j\widetilde{\varepsilon}_b^j, \tag{41}$$

with parameters $\gamma_j \geq 0$ for $j = 1, ..., m$. Combining (40) and (41), one finds

$$\widetilde{v}_b' = \widetilde{z} - \delta' \widetilde{\varepsilon}_b^{k+1}, \tag{42}$$

where $\widetilde{z}$ is a random variable independent from $\widetilde{\varepsilon}_b^{k+1}$, and

$$\delta' = \gamma_{k+1} - (1+h')\mu_{k+1}\delta = \gamma_{k+1} - \frac{(1+h)\mu_{k+1}\delta}{1-(\mu_{k+1}/\mu_k - 1)\delta}. \tag{43}$$

As $\partial\delta'/\partial\delta < 0$, it suffices to show that $\partial E[\widetilde{u}_L]/\partial\delta' < 0$, where the derivative is evaluated at $\delta' = \gamma_{k+1}$. Let $G(.)$ and $H(.)$ denote the distribution functions of random variables $\widetilde{z}$ and $\widetilde{\varepsilon}_b^{k+1}$, respectively. Then, using (42), expected utility (1) for the lender at the time of contracting reads

$$E[\widetilde{u}_L] = \pi_G u_L(r) + \pi_B \iint u_L(\min\{z - \delta'\varepsilon_b^{k+1} - 1; r\}) dH(\varepsilon_b^{k+1}) dG(z), \tag{44}$$

where z and ε_b^{k+1} denote the realizations of random variables $\widetilde{z}$ and $\widetilde{\varepsilon}_b^{k+1}$, respectively. The weak inequality $\partial E[\widetilde{u}_L]/\partial\delta' \leq 0$ follows from standard arguments (cf., e.g., Tesfatsion, 1976), but the strict inequality requires a proof. For this, note that the interior integral in (44) reads

$$\begin{aligned} E[\widetilde{u}_L|B,z] &= \int u_L(\min\{z - \delta'\varepsilon_b^{k+1} - 1; r\}) dH(\varepsilon_b^{k+1}) && (45) \\ &= u_L(r) H(\frac{r-z+1}{\delta'}) + \int_{\frac{r-z+1}{\delta'}}^{\infty} u_L(z - \delta'\varepsilon_b^{k+1} - 1) dH(\varepsilon_b^{k+1}), && (46) \end{aligned}$$

and can be differentiated with respect to δ' at $\delta' = \gamma_{k+1}$. We obtain

$$\begin{aligned} \frac{\partial}{\partial\delta'} E[\widetilde{u}_L|B,z] &= -\mu_{k+1} \int_{\frac{r-z+1}{\delta'}}^{\infty} \varepsilon_b^{k+1} u_L'(z - \delta'\varepsilon_b^{k+1} - 1) dH(\varepsilon_b^{k+1}) && (47) \\ &\leq -u_L'(r)\mu_{k+1} \int_{\frac{r-z+1}{\delta'}}^{\infty} \varepsilon_b^{k+1} dH(\varepsilon_b^{k+1}), && (48) \end{aligned}$$

where the inequality follows from the fact that $u_L'(.)$ is weakly declining. Now, by Assumption 5,

$$E[\widetilde{\varepsilon}_b^{k+1}] = \int_{-\infty}^{\infty} \varepsilon_b^{k+1} dH(\varepsilon_b^{k+1}) > 0, \tag{49}$$

so that $\partial E[\widetilde{u}_L|B,z]/\partial\delta' \leq 0$. It suffices to show that $\partial E[\widetilde{u}_L|B,z]/\partial\delta' < 0$ is strict for "sufficiently many" z. Note that Assumption 5 implies that collateral is imperfect. From Theorem 1 and efficiency, $\mathrm{pr}\{\widetilde{v}_b < 1+r\} > 0$. Thus, by (42) and independence,

$$\mathrm{pr}\{\widetilde{v}_b \geq 1+r\} = \int H(\frac{1+r-z}{\gamma^{k+1}}) dG(z) < 1. \tag{50}$$

Therefore, there must be a compact interval Z satisfying $\int_Z dG(z) > 0$ such that for any $z \in Z$, we have $H(\frac{1+r-z}{\gamma^{k+1}}) < 1$. Fix $z \in Z$. From (49) and $H(\frac{1+r-z}{\gamma^{k+1}}) < 1$, clearly

$$\int_{\frac{1+r-z}{\gamma^{k+1}}}^{\infty} \varepsilon_b^{k+1} dH(\varepsilon_b^{k+1}) = (1-H(\frac{1+r-z}{\gamma^{k+1}}))E[\widetilde{\varepsilon}_b^{k+1}|\widetilde{\varepsilon}_b^{k+1} \geq \frac{1+r-z}{\gamma^{k+1}}] \quad (51)$$

$$\geq (1-H(\frac{1+r-z}{\gamma^{k+1}}))E[\widetilde{\varepsilon}_b^{k+1}] > 0, \quad (52)$$

so that by (48), we find indeed that $\partial E[\widetilde{u}_L|B,z]/\partial\delta' \leq 0$ for all $z \in Z$. Hence, $\partial E[\widetilde{u}_L]/\partial\delta' < 0$. Thus, for small enough $\delta > 0$, the lender's expected utility at the time of contracting is strictly increasing in δ. Clearly, the borrower's expected utility at the time of contracting is weakly increasing with a change from C to $C'(\delta)$. Thus, the initial SRA C cannot be efficient. $\square$

Proof of Theorem 3. Define r^B, r^D, h_0 as in Lemma A.1 below. Then $r^B > r_0 > r^D$. Moreover, for any haircut $h \geq -1$, either $h < h_0$ or $h \geq h_0$. If $h < h_0$, then $\rho^D(h) \geq \rho^D(h_0) > r^B \geq \rho^B(h)$, so there is no repo rate for which the market transaction is individually rational for lender and borrower at the same time. If $h \geq h_0$, then $\rho^B(h) \leq \rho^B(h_0) < r^D \leq \rho^D(h)$, and again no market transaction is feasible. $\square$

Lemma A.1. *There is a haircut $h_0 \geq -1$ and interest rates r^B, r^D satisfying $r^B > r_0 > r^D$ such that $\rho^D(h_0) > r^B$ and $\rho^B(h_0) < r^D$.*

Proof. As collateral is imperfect, there is a cut-off price p^* such that $F_b(p^*) > 0$ and $F_a(p^*) < 1$. Define the haircut h_0 by $p^* = (1+r_0)/(1+h_0)$. Let $r^D = r_0 - \varepsilon$ and $r^B = r_0 + \varepsilon$ for $\varepsilon > 0$ small. It will be shown that for ε small enough, $\rho^D(h_0) > r^B$ and $\rho^B(h_0) < r^D$. By the definition of $\rho^D(h_0)$,

$$\begin{aligned}(\pi_G + \pi_B)u_L(r^D) &= (\pi_G + (1-F_b(p_b^*))\pi_B)u_L(\rho^D(h_0)) \\ &\quad +\pi_B \int_{p_b \leq p_b^*} u_L((1+h_0)p_b - 1)dF_b(p_b),\end{aligned} \quad (53)$$

where $p_b^* = (1+\rho^D(h_0))/(1+h_0)$. Re-arranging (53) yields

$$\begin{aligned}u_L(\rho^D(h_0)) &= u_L(r^D) \\ &\quad +\frac{\pi_B}{\pi_G + F_b(p_b^*)\pi_B} \int_{p_b \leq p_b^*} (u_L(r^D) - u_L((1+h_0)p_b - 1))dF_b(p_b),\end{aligned} \quad (54)$$

where the integral is either positive or zero. To provoke a contradiction, assume that

$\rho^D(h_0) \le r^B$ for all small $\varepsilon > 0$. Then $p_b^* \le \widehat{p}_b = (1+r^B)/(1+h_0)$, and consequently,

$$\begin{aligned} & u_L(\rho^D(h_0)) \\ \ge\ & u_L(r^D) + \frac{\pi_B}{\pi_G + F_b(\widehat{p}_b)\pi_B} \int_{p_b \le p_b^*} (u_L(r^D) - u_L((1+h_0)p_b - 1)) dF_b(p_b). \end{aligned} \tag{55}$$

For $p_b < \widehat{p}_a = (1+r^D)/(1+h_0)$, the expression integrated in (55) is positive, while for $p_b \ge \widehat{p}_a$, the expression is negative or zero. Hence, splitting the integral yields

$$\begin{aligned} & u_L(\rho^D(h_0)) \\ \ge\ & u_L(r^D) + \frac{\pi_B}{\pi_G + F_b(\widehat{p}_b)\pi_B} \int_{p_b < \widehat{p}_a} (u_L(r^D) - u_L((1+h_0)p_b - 1)) dF_b(p_b) \\ & - \frac{\pi_B}{\pi_G + F_b(\widehat{p}_b)\pi_B} \int_{\widehat{p}_a \le p_b \le p_b^*} (u_L((1+h_0)p_b - 1) - u_L(r^D)) dF_b(p_b) \qquad (56) \\ \ge\ & u_L(r^D) + \frac{\pi_B}{\pi_G + F_b(\widehat{p}_b)\pi_B} \int_{p_b < \widehat{p}_a} (u_L(r^D) - u_L((1+h_0)p_b - 1)) dF_b(p_b) \\ & - \frac{\pi_B}{\pi_G + F_b(\widehat{p}_b)\pi_B} \int_{\widehat{p}_a \le p_b \le \widehat{p}_b} (u_L((1+h_0)p_b - 1) - u_L(r^D)) dF_b(p_b) \qquad (57) \\ \ge\ & u_L(r^D) + \frac{\pi_B}{\pi_G + F_b(\widehat{p}_b)\pi_B} \int_{p_b < \widehat{p}_a} (u_L(r^D) - u_L((1+h_0)p_b - 1)) dF_b(p_b) \\ & - \frac{\pi_B}{\pi_G + F_b(\widehat{p}_b)\pi_B} \int_{\widehat{p}_a \le p_b \le \widehat{p}_b} (u_L(r^B) - u_L(r^D)) dF_b(p_b). \qquad (58) \end{aligned}$$

For $\varepsilon \to 0$, we would have $\rho^D(h_0) \to r_0$, and therefore in the limit

$$\begin{aligned} & u_L(\rho^D(h_0)) \ge \\ & u_L(r^D) + \frac{\pi_B}{\pi_G + F_b(p^*)\pi_B} \int_{p_b < p^*} (u_L(r^D) - u_L((1+h_0)p_b - 1)) dF_b(p_b). \end{aligned} \tag{59}$$

For any values $\widehat{p}_b, \widehat{p}_a$ sufficiently close to p^* it is still true that $F_a(\widehat{p}_b) < 1$ and $F_b(\widehat{p}_a) > 0$. In particular, the integral in (59) is strictly positive. Using Assumption 1, we find a contradiction to the assumption that $\rho^D(h_0) \le r^B$ for all small $\varepsilon > 0$. Thus, $\rho^D(h_0) > r^B$ for *some* sufficiently small ε. But for decreasing ε, the interest rate r^B is decreasing, while r^D is increasing so that $\rho^D(h_0)$ is non-decreasing. Hence, $\rho^D(h_0) > r^B$ for *any* sufficiently small ε. An analogous argument can be used to show that also $\rho^B(h_0) < r^D$ for all sufficiently small ε. □

Proof of Theorem 4. Assume that (θ^1, θ^2) is stable. To provoke a contradiction, assume that by at least one counterparty, some collateral k is placed with the central bank, but collateral $k+1 \le m^{CB}$ is not at all or not exclusively used with the central bank. Formally, there is some i_B such that $\theta_k^{i_B} > 0$ and $(1+\eta_{k+1})\theta_{k+1}^{i_B} D_{i_B} < q_{k+1}^{i_B}$. Let λ be such that

$$\sum_{j=1}^{m_{\text{CB}}} q_j^{i_B} \overline{p}_a^j > \lambda > \sum_{j=1}^{k} (q_j^{i_B} - (1+\eta_j)\theta_j^{i_B} D_{i_B}) \overline{p}_a^j. \tag{60}$$

As $\theta_k^{i_B} > 0$, such λ clearly exists. By Assumption 6, there is no market imperfection for λ. Hence, given that (θ^1, θ^2) is stable, there is an efficient true SRA between borrower and lender. Following now the lines of the proof of Theorem 2, it can be seen that the lender (borrower) can strictly (weakly) gain for this given λ if the borrower replaces a small quantity of collateral k deposited with the central bank by a corresponding quantity of collateral $k+1$. Hence (θ^1, θ^2) cannot be stable. $\square$

Proof of Theorem 5. Fix $J' \supseteq J$, and $\lambda > 0$. Assume first that policy (J, α) does not admit a market transaction. Then lender and borrower obtain their outside option utilities $\underline{u}_L = (\pi_G + \pi_B) u_L(r^D)$ and $\underline{u}_B(J) = (\pi_G + \pi_B) u_L(r^B(J))$, respectively. Choose $\alpha' = \alpha$. If there is also a market imperfection under policy (J', α'), then $\underline{u}_B(J') \geq \underline{u}_B(J)$ increases weakly, while $\underline{u}_L$ remains unchanged. If, however, a market transaction comes about under policy (J', α'), then by individual rationality, $E[\widetilde{u}_L | (J', \alpha')] \geq \underline{u}_L$ and $E[\widetilde{u}_B | (J', \alpha')] \geq \underline{u}_B(J') \geq \underline{u}_B(J)$. Thus, either way, there is a weak Pareto improvement. Assume now that a market transaction comes about under policy (J, α). Then the weak enlargement of the set of eligible collateral implies a weak enlargement of the bargaining set, and a weak increase in the borrower's outside option utility from $\underline{u}_B(J)$ to $\underline{u}_B(J')$. Clearly, $E[\widetilde{u}_L | (J, \alpha)] \geq \underline{u}_L$. Moreover, by assumption, $E[\widetilde{u}_B | (J, \alpha)] \geq \underline{u}_B(J')$. Therefore, the pair of expected utilities $(E[\widetilde{u}_B | (J, \alpha)], E[\widetilde{u}_L | (J, \alpha)])$ is individually rational under collateral policy J' and contained in the bargaining set for collateral policy J'. Noting that the bargaining set for policy J' is convex (possibly as a result of efficient randomization over SRAs), there is a liquidity policy α' such that Nash bargaining implies a weak Pareto improvement for market participants by switching from policy (J, α) to (J', α'). $\square$

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Eurosystem (Source: ECB data)		Repo market (Source: ICMA)	
Central Gov	22.5%	EU Central Gov	84.2%
Regional Gov	6.6%	Other EU	15.8%
Uncov Bank Bonds	31.5%	Total	100.0%
Cov Bank Bonds	13.2%		
Corporates	6.6%		
ABS	12.1%		
Other marketable	3.8%		
Credit claims	3.7%		
Total	100.0%		

Table I. Average Collateral Usage during 2006 in Primary and Secondary Funding

The entries on the left-hand side refer to market values of assets, net of haircuts, held as collateral by counterparties with the Eurosystem as an average of monthly data (end of business, last Friday of the month) for 2006. Shown are the percentage shares of different types of assets eligible as collateral with the Eurosystem. The abbreviations "Central Gov" and "Regional Gov" refer, respectively, to central government bonds such as issued, e.g., by the Federal Republic of Germany, and to regional government bonds such as issued, e.g., by the Federal State of Hesse. Similarly, "Uncov Bank Bonds" and "Cov Bank Bonds" should be read as uncovered and covered bank bonds, respectively. Vis-à-vis the Eurosystem, collateral can be held either through a pooling or through an earmarking system. Earmarking has been used predominantly in France, Italy, and Ireland. There are some countries where both collateral systems are in use. Most national central banks rely exclusively on the pooling system. In a pooling system, collateral assets may exceed the outstanding credit (i.e., there may be over-collateralization). The entries on the right-hand side represent percentage shares of different types of EU collateral used in the euro repo market. Reported are averages over values reported by 79 (74) financial units as outstanding at close of business for June 14, 2006 and December 13, 2006.

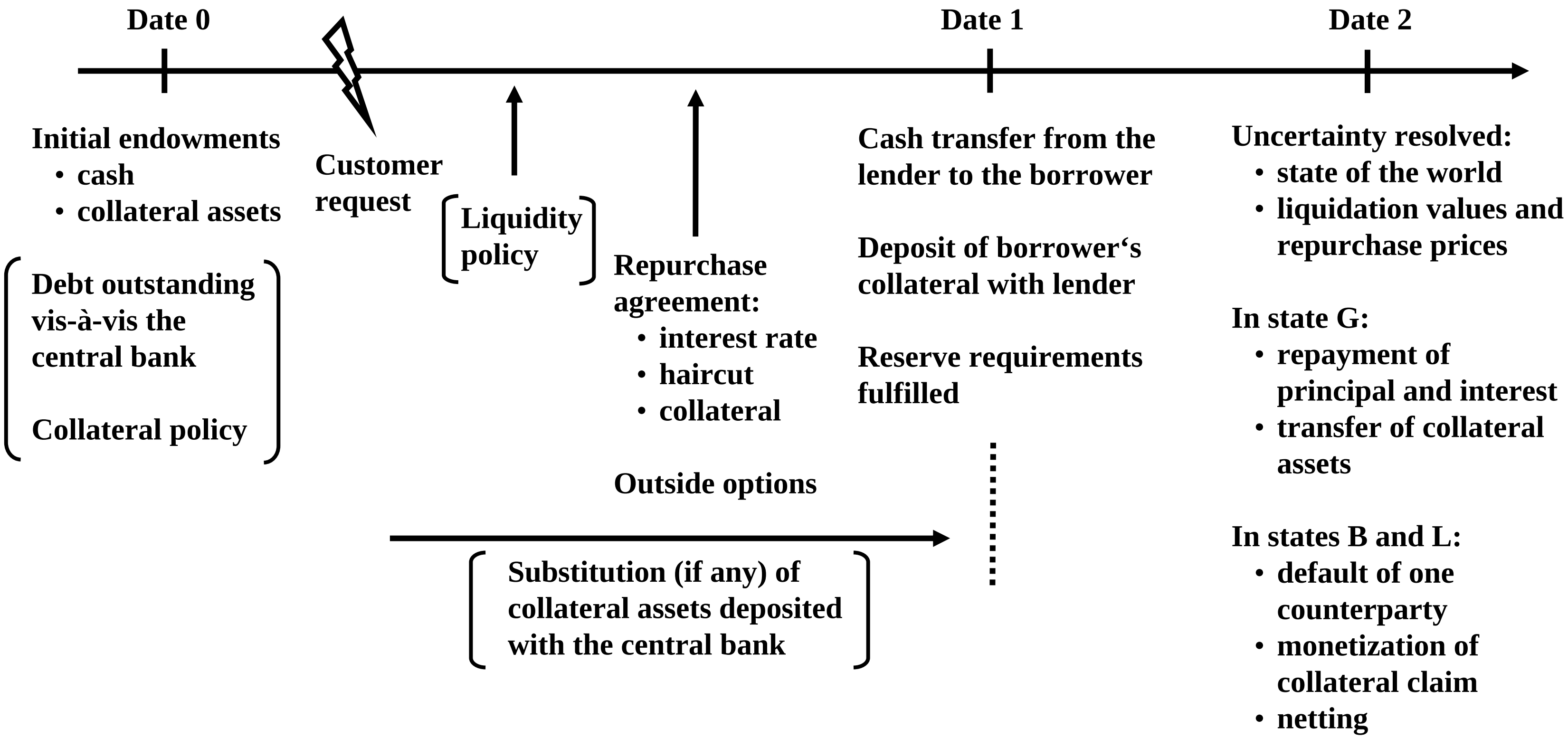

Figure 1. Time Structure of the Model

The figure exhibits the time structure of the full-fledged repo model. For the basic analysis performed in Sections I through III, items set in brackets should be ignored. At date 0, each of the two banks is endowed with cash and collateral, and holds a debt position vis-à-vis the central bank. Also at date 0, the central bank decides about the range of assets accepted as collateral between dates 1 and 2. Between dates 0 and 1, a liquidity shock in the form of a random customer request affects the expected cash position of the banks, and assigns a role to each of them as either cash lender or cash borrower in the interbank market. The central bank reacts to the liquidity shock by choosing a liquidity policy. Consequently, the banks negotiate about the conditions of the repurchase agreement, taking account of their respective outside options. Substitution of collateral assets placed with the central bank is feasible at any time between the customer request and date 1. The value date of the repurchase agreement is date 1, which is also the date at which the fulfillment of minimum reserve requirements is controlled. At the terminal date 2, the uncertainty about the state of the world is resolved, and liquidation values and replacement costs become common knowledge. In the good state G, the repurchase agreement is terminated according to the regular terms of the contract. In case of unilateral default by either the cash borrower (state B) or the cash lender (state L), the lender's claim on the collateral is monetized and netted with the claim on repayment of principal and interest, potentially leaving the non-defaulting party with a residual loss.

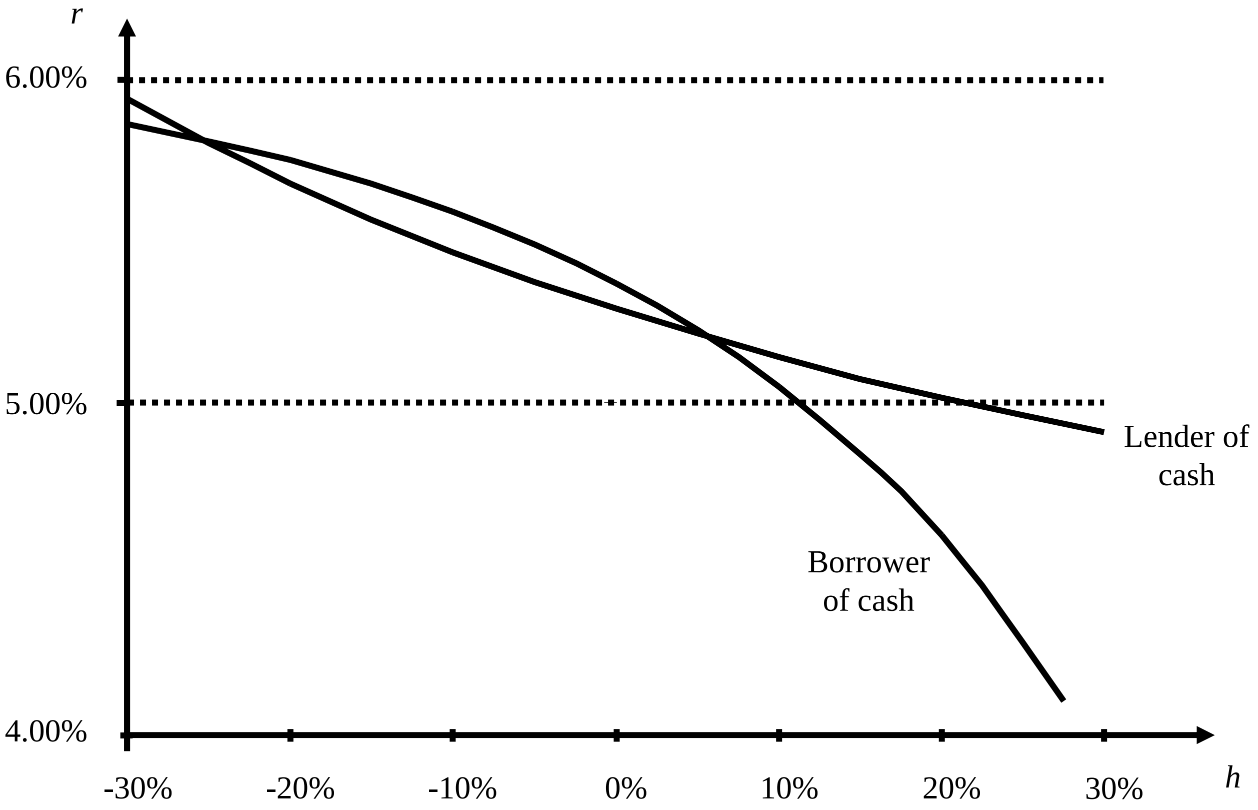

Figure 2. Indifference Curves of Cash Lender and Cash Borrower

The two graphs in the figure show, respectively, the minimum acceptable repo rate for the lender of cash and the maximum acceptable repo rate for the borrower of cash, both as a function of the haircut. In this numerical example, utility functions of the counterparties have each a constant coefficient 1 of absolute risk aversion. The respective conditional distributions for liquidation and repurchase values are identical and of the Erlang-2 type with mean 1. Probabilities of default are 1% for the cash lender and 3% for the cash borrower. The opportunity borrowing rate is 6%, the opportunity deposit rate is 4%. An agreement can be seen to be individually rational simultaneously for both counterparties for haircuts in the range between about -25% through about +5%. The example illustrates the possibility of negative haircuts even when the cash lender has the higher default probability than the cash borrower. The effect is caused by the right-skewness of the Erlang distribution, which exposes especially the cash borrower to non-negligible counterparty risk.