Renegotiating Distressed Mortgage Loans: A Structural Estimation*

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preliminary and incomplete

Abstract

This paper builds and estimates a dynamic equilibrium model to study renegotiation between borrowers and lenders over distressed fixed-rate mortgage loans using a unique mortgage loan level data. The model captures borrowers' as well as lenders' behavior prior to borrowers' mortgages becoming distressed and after mortgage renegotiation. It allows for private information regarding borrowers' payment preference and features various realistic transaction costs associated with mortgage modification and foreclosure. Our preliminary estimation of borrowers' decision given lenders' which is estimated from the model reveals that there exists substantial moral hazard as borrowers respond strongly to their perception of modification probability. Additionally, private information also appears to be important as it is featured prominently in borrowers' payment decisions.

JEL Classification: D1, D8, G2

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1 Introduction

The meltdown of US residential house prices commenced in 2006 has led to the worst mortgage default and foreclosure crisis since the Great Depression. Many researchers and policy makers have attributed the severity of the crisis to the lack of mortgage modifications following mortgage defaults. Policy makers have also devised various prevention plans in response to the crisis with a focus on loan modifications.[1] Despite the prominent role given to mortgage renegotiation in the policy debate and the growing literature on mortgage default, there has been no consensus as to what caused the lack of renegotiation and no framework where various government policies can be evaluated.

This paper fills in this gap. We first develop a dynamic equilibrium model to study the behavior of borrowers as well as lenders before, during and after mortgage renegotiation and in the presence of income and house price uncertainties. The model features private information as lenders do not observe borrowers' payment aversion and allows for various realistic transaction costs associated with mortgage default and foreclosure for borrowers and lenders.

In the second step, we estimate the model using a unique mortgage loan level data where the majority of mortgage modifications are directly identified rather than inferred as in many of the previous analysis. We further merge the data with Home Mortgage Disclosure Act (HMDA) to obtain borrowers' income at mortgage origination and some demographic information, and with consumer credit bureau data to obtain information on borrowers' other liabilities. Our preliminary estimation of borrowers' problem given lenders' decisions estimated from the data indicates that borrowers' perception of modification probability plays an important role in their payment decisions. In other words, there exists moral hazard. Secondly, private information is featured prominently in borrowers' decision suggesting that it may also accounts for lenders' reluctance to modify delinquent mortgages.

Using our estimated framework, we conduct several policy analysis. ....

The paper contributes to a growing area of research in foreclosure loss mitigation associated with residential mortgages. On the theoretical front, Ambrose and Capone (1996) are one of the first to formalize the cost-benefit analysis of the lender's decision to either foreclose or renegotiate with a seriously delinquent borrower. They find that self-cure risk – a situation in which a delinquent borrower is able to catch up on their delinquent mortgage payment without any help from the lender – is a very important component of the cost-benefit analysis. Riddiough and Wyatt (1994) are the first to analyze the foreclosure versus

[1] For example, in October 2007, then-Treasury Secretary Henry Paulson announced the creation of the Hope Now Alliance. Hope Now provides to homeowners loan workouts that result in establishing either a repayment plan with the homeowner to bring them back to current or a permanent loan modification where the terms of the the mortgage are modified (see Gerardi and Li 2010 for an overview of mortgage foreclosure prevention efforts).

renegotiation decision in a strategic environment in which lenders hold private information regarding their cost of foreclosure. Lenders' decision of foreclosure versus renegotiation thus acts as a signal of their foreclosure cost and borrowers make their payment decisions based on their assessment of the cost. Wang, Young, and Zhou (2002) build up on Riddiough and Wyatt (1994) and argue that the existence of asymmetric information between borrowers and lenders suggests that it is optimal for lenders to randomly reject concessionary modification requests and the rejection rates depend on the cost of foreclosure to lenders, the benefits of delinquency to borrowers, and the fraction of delinquent borrowers in the economy. All the models are static and abstract from the impact of future house price movement and income fluctuations on renegotiation outcomes.[2]

Compared with the theoretical literature, the empirical literature has focused largely on addressing the question of why there have been few concessionary modifications to distressed mortgages, particularly the role of mortgage securitization and the presence of private information. A servicer is likely to internalize benefits of modifying securitized mortgages. Additionally, the pooling and servicing agreements may also restrict servicers' latitude in offering modifications. Private information, on the other hand, leads to mortgage hazard and adverse selection, and thus increase the cost of loan modification for lenders. Adelino, Geradi, and Willen (2009) and Priskorski, Seru, and Vig (2009) both study whether mortgage securitization led to less than efficient levels of mortgage renegotiations. While Priskorski et al. (2009) find supporting evidence that securitization led to less mortgage modification, Adelino et al. (2009) do not find any supporting evidence.[3] Agarwal, Amromin, Ben-David, Chomsisengphet, and Evanoff (2012) document the lack of mortgage modification during the early phrase of the crisis and confirm Adelino et al. (2009)'s finding that securitized loans are less likely to be modified. Mayer, Morrison, Piskorski, and Gupta (2011) test the possibility that homeowners respond strategically to news of mortgage modification programs by exploiting plausibly exogenous changes in modification policy induced by government lawsuits and find that strategic behavior is an important consideration in designing mortgage modification programs.

The rest of the paper is organized as follows. Section 2 discusses the data, section 3 builds the model. Section 4 describes the estimation strategy and section 5 presents the estimation results. Section 6 concludes.

[2]Recently, Adelino, Gerardi, and Willen (2009) develop a simple model allowing future prices to affect lenders' decision between modification and foreclosure. Their analysis, however, ignores borrowers' strategic behavior.

[3]See Gerardi and Li (2010) for a detailed discussion of the differences between the two papers.

2 Data

2.1 Data Sources

Our main data are the CoreLogic Private Label Securities database – ABS and the CoreLogic Loan Modification database. The CoreLogic ABS database consists of loans originated as subprime and Alt-A loans and represents the most complete set of such loan level data available with a coverage rate of over 90 percent. The data include loan level attributes generally required of issuers of these securities when they originate the loans as well as historical performance and is updated monthly. These attributes include borrower characteristics (credit scores, owner occupancy, documentation type and loan purpose); collateral characteristics (mortgage loan-to-value ratio, property type, zip code); and loan characteristics (product type, loan balance, and loan status). The CoreLogic Loan Modification database contains information on modifications done to loans in the CoreLogic ABS database. It contains detailed information about modified terms. The merge of the two data sets are straightforward as a loan is uniquely identified by the same loan id in both databases.

The third data source is the Home Mortgage Disclosure Act (HMDA). HMDA contains information of almost all mortgage originations in the U.S. and provides demographic information such as homeowners' income, race, sex, and marital status at the time of the mortgage application. We match the CoreLogic database with HMDA by linking information on the location of the property and several important characteristics of the mortgage.[4] The final data set is the Federal Reserve Bank of New York (FRBNY) Consumer Credit Panel/Equifax. The panel represents a nationally representative 5% random sample of consumers drawn from Equifax credit report data and their household members. In all, the sample is about 15 percent of the original Equifax credit report data.[5] It contains quarterly information on an individual's borrowing and payment of various loans (loans from bank cards and department store cards, car loans, mortgages, home equity loans, etc.) as well as some demographic information such as the individual's age and address. The match of the CoreLogic data with Equifax is conducted similarly as that of CoreLogic and HMDA.[6]

Our thus constructed data have several advantages over most of those used in the literature. First, with the exception of Agarwal et al. (2010), all the existing studies have to infer mortgage modifications. In our data, over 50% of the mortgage modifications are

[4] The merge procedure is similar to that described in Haughwout, Mayer, and Tracy (2009). Mortgages were matched based on the zip code of the property, the date when the mortgage was originated (within five days), the origination amount (within $1000), the purpose of the loan (purchase, refinance, or other), the type of loan (conventional, VA guaranteed, FHA guaranteed, or other), occupancy type (owner-occupied or non-owner-occupied), and lien status (first lien or other). The match rates for loans originated between 2004 and 2006 are close to 60%.

[5] Equifax is one of the four major credit reporting agencies.

[6] The match rate averages about 10% for loans originated between 2004 and 2006.

directly provided by loan servicers to CoreLogic and the rest are inferred by CoreLogic professionals based on information on changes in loan balances, interest rates, maturity and other loan characteristics. Second, the match with HMDA provides us with demographic information such as income at origination, marital status, and homeowners' age that are important for homeowners' mortgage payment decisions. Finally, the match with FRBNY Consumer Credit Panel/Equifax allows us to capture all the liabilities of homeowners' as well as the performance of these liabilities. This information is also important for explaining homeowners' mortgage payment decision.

2.2 Data Description

We take the matched first lien fixed rate mortgage loans originated between 2004 and 2007 from the four crisis states, Arizona, California, Florida, and Nevada.[7] The sample spans the period between January 2007 to February 2009 and the loans have to be 30 days or more delinquent at some point during the sample period to be included. We chose this study period so that we observe a nontrivial number of mortgage modifications without government intervention.[8]

In total, we have 4,959 unique mortgage loans. Of these loans, 21 percent were originated in 2004, 33 percent in 2005 and 2006, respectively, and 12 percent in 2007. A little over half of the loans (51 percent) were for properties located in Florida, 38 percent in California, 8 percent in Arizona, and 4 percent in Nevada. Finally, about 4.5 percent of the mortgage loans were modified with Florida having the highest modification rates followed by California and then Arizona and Nevada. The majority of the mortgage modifications occurred in 2008 (71 percent), some in 2009 (21 percent), and a few in 2007 (8 percent).

We then merge our data with CoreLogic monthly zip code level house price index and county unemployment rates from the Bureau of Labor Statistics. The final data have over 110,000 observations. Table 1 presents the summary statistics of our whole sample and of the modified sample. The noticeable differences between the two samples are, borrowers of modified sample have on average lower income and higher mortgage loan-to-value ratios at origination. Their current mortgage interest rates tend to be higher and so is remaining principal balance. They also tend to live in areas with relatively higher unemployment rates. Most strikingly, borrowers at the modified sample have a much higher 60 days or more delinquency rates and much higher foreclosure rates including foreclosure start, real estate own, and liquidation.

[7] We intend to expand the data set to include more states and adjustable rate mortgages in later versions.

[8] Mortgage delinquency rates start to increase dramatically in 2007. The first coordinated large-scale government effort to modify mortgage loans – the "Making Home Affordable" program was unveiled in February 2009.

Table 1. Summary Statistics

variable	whole sample			modified loans		
	mean	median	s.d.	mean	median	s.d.
borrower age (years)	48	47	12	47	46	12
with co-applicant	0.52	1	0.50	052	1	0.50
income at origination ($1000)	88.86	69.00	107.58	74	64	43
risk score	651	666	114	590	578	123
loan amount at orig. ($1000)	243	201	156	228	200	125
ltv ratio at orig.	73	79	14	78	80	12
current interest rate (%)	7.00	6.75	1.03	7.39	7.25	1.43
principal balance	236	196	153	226	199	124
age of the loan (months)	27	26	13	25	24	13
remaining terms (months)	334	335	13	336	337	13
60 days or more delinquent	0.108	0.000	0.310	0.316	0.000	0.465
foreclosure start and liquidation	0.074	0.000	0.195	0.187	0.000	0.390
local unemployment rates (%)	6.21	5.60	2.50	6.31	5.70	2.42
number of observations	112,006			5,461		

Turning to the modified loans, at the time of modification, the average age of the loan is 30 months and the median age is 29 months. Figure 1 depicts the histogram of loan age at the time of modification. For the fixed rate mortgages that we examine here, we observe only three types of mortgage modification, principal forgiveness, interest rate reduction, and recapitalization. However, principal reduction constitutes only 3 percent of the total mortgage loan modification. To ease our computation, we exclude these loans from our sample. As a result, we can model the rest of the modification simply as an interest rate reduction that may or may not result in a larger principal balance (recapitalization). The average rate of interest rate reduction is 1.41 percentage points and the median is 0.90 percentage points. About 45 percent of the modifications, however, result in no change in interest rate. These findings are consistent with Agarwal, Amromin, Ben-David, Chomsisengphet, and Evanoff (2012). We chart the distribution of interest rate reduction in Figure 2. In terms of mortgage balance, after the modification, mortgage principal balance increases, on average, by $5,764 with a median of $4,324. After the modification, the monthly payments come down by, on average, $126, and the median reduction in monthly payments is $47. For some homeowners, the monthly payment actually goes up as these homeowners experience recapitalized most or all of their arrearage without receiving much reduction in interest rates.

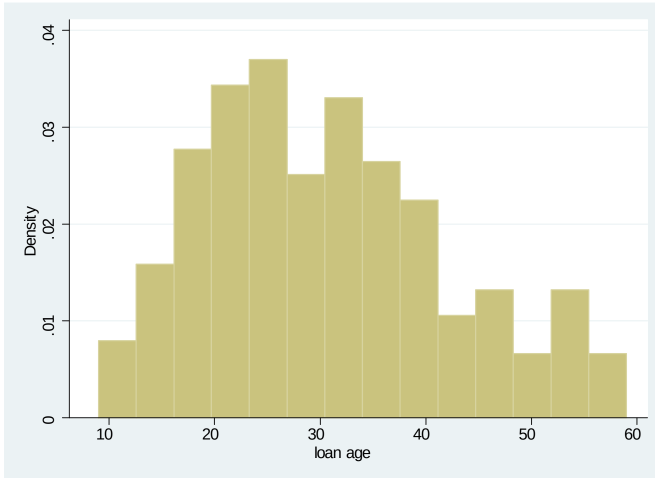

Figure 1. Distribution of Loan Age (months) at Modification

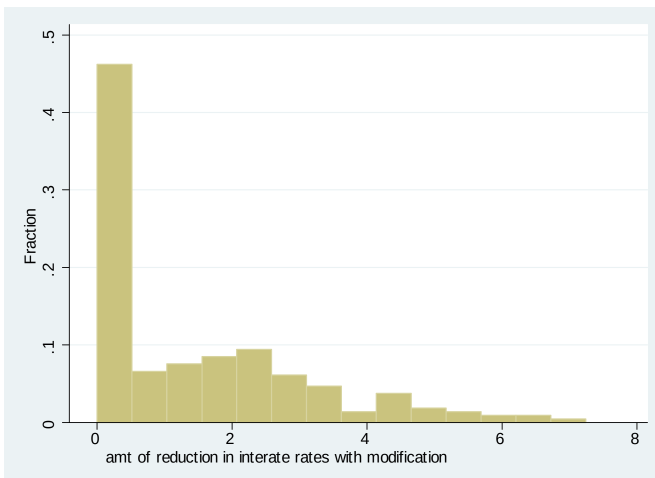

Figure 2. Distribution of Interest Rate Reduction at Modification

With regard to mortgage performance, at the time of modification, 85 percent of the loans are 60 days or more delinquent and the rest 15 percent are 30 days delinquent. After the modification within our sample period, 30 percent of the loans become 60 days or more delinquent again and 17 percent enter into foreclosure.

3 Model

We model a dynamic interaction between a borrower and lender, who are linked by a mortgage loan. Time is discrete and finite with each period representing one month. The borrower's mortgage is a fixed rate mortgage loan of maturity T, balance bal_0, and interest rate r_0. The borrower and the lender face uncertainties: local house prices h_t and unemployment rates unr_t, which all fluctuate over time according to exogenously given stochastic processes.

Each player (the borrower and lender) maximizes the expected sum of discounted future payoffs in each time. the borrower is described by his permanent unobserved discrete type $s \in S$ that affects his willingness to pay. We assume that s is not directly observed by the lender or econometrician. However, the distribution for type s depends on the borrower's initial characteristics, which is included in the observed state. With the exception of the borrower's type s, the lender observes all other information perfectly.

The timeline is as follows. Each period, after all uncertainties are resolved, the borrower makes his payment decision taking into consideration the lender's response. Based on the borrower's payment decision, the lender acts accordingly. Consistent with reality, we assume that the lender will take action only when the borrower defaults on his mortgage. Figure 3 depicts the decision tree. In the sections that follow, we describe the borrower and lender's decisions in details.

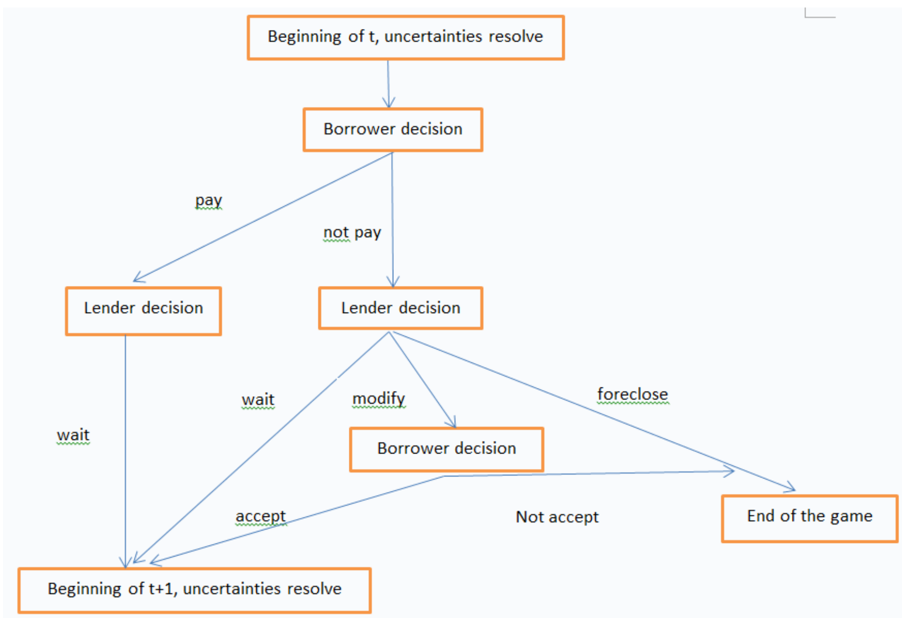

Figure 3. Decision Tree

3.1 Period Utility

3.1.1 The Borrower

Choice Set Let us denote the borrower's required period payment by mp. In each period t, if the borrower has been current on his mortgage payment, then he makes the decision of whether to make the payment mp or to not pay.[9] If the borrower has missed a payment,

[9] The borrower usually has an option to pay off (or prepay) the entire loan through refinancing or selling, which will end the current relationship between the borrower and lender. However, we do not allow for the option in our model because the option is rarely chosen by the borrower in a given period (with probability less than 0.2%) in our sample.

then he makes the following decisions, paying just mp and staying in a status of one-month-delinquency, paying $2mp$ to be current again, or not paying. To generalize, if the borrower has δ unpaid monthly payments at the beginning of time t, we assume that for $\delta < 3$ he makes the following decisions: paying $mp, \cdots, (\delta+1)mp$, or no payment. For $\delta \geq 3$, we assume that he only has the option of paying $(\delta-1)mp$, $(\delta)mp$, or $(\delta+1)mp$ to become two-month delinquent, one-month delinquent, or current, respectively. He also has an option of not paying.[10]

Formally, let $A_b(\delta_t)$ denote the choice set of the borrower at time t, where δ_t denote the number of months for which the borrower is delinquent. If the borrower is current on his mortgage, $\delta_t = 0$. The borrower's choice set $A_b(\delta_t)$ is:

$$A_b(\delta_t) = \begin{cases} \{0, mp\}, & \text{if } \delta_t = 0; \\ \{0, mp, 2mp\}, & \text{if } \delta_t = 1; \\ \{0, (\delta_t - 1)mp, \delta_t mp, (\delta_t + 1)mp\}, & \text{if } \delta_t \geq 2; \end{cases}$$

where $a_{bt} \in A_b(\delta_t)$ represents an action that the borrower pays a_{bt} dollars to the lender. In particular, that $a_{bt} = 0$ means not making any payments.

Utility Let $x_t \in X$ denote the state of the economy in period t after all uncertainties are resolved. This state is observed by both players and the econometrician. In each period, the borrower also receives a preference shock denoted as $\xi_t \sim G(x_t)$, which is also private information to the borrower along with his type s. We assume that the borrower of type s in state x_t with preference shock ξ_t receives (per-period) indirect utility of $u_b(a_{bt}; x_t, s, \xi_t)$ from action $a_{bt} \in A_b(\delta_t)$:

$$\begin{aligned} u_b(a_{bt}; x_t, s, \xi_t) = {} & \beta_0 + \beta_h h_0 + \beta_{pay}\mathbf{1}[a_{bt} > 0] + \beta_{crnt}\mathbf{1}[\delta_t = 0] \\ & + (\beta_{eq,1} + \beta_{eq,2}\mathbf{1}[\delta_t \geq 3])(h_t - bal_t) - (\alpha_s + v_t)(a_{bt} + \alpha_{sq} a_{bt}^2) + \epsilon_t(a_{bt}) \end{aligned}$$

where $\xi_t = (v_t, \epsilon_t)$. Note that since the lender moves after the borrower in period t, the lender's action will not directly affect the borrower's period utility but continuation payoffs in the future.

The first two terms in the utility capture a consumption value of living in a house, which

[10] This assumption on choice sets is due to data restriction. In the data, we do not directly observe actual mortgage payment in a given month. We have to infer it from changes in loan status. The loan status, however, only records whether the loan is current, 30 days delinquent, 60 days delinquent, 90 days or more delinquent, and in foreclosure. In our conversations with CoreLogic experts, we were told that lenders typically do not distinguish much between whether the borrower is 90 days delinquent, 120 days delinquent, or more.

depends on the initial house price at the time of mortgage origination (h_0).[11,12] Next, there is a constant for actions with positive payments (β_{pay}); the next two terms β_{crnt} and β_{sd} capture (dis)utility from being current and seriously delinquent in a mortgage. A borrower's credit score will suffer once he becomes delinquent on his debt. As a result, he will have difficulty accessing credit in the future. Some lenders may not lend to him and others may lend less to him and at higher rates.

We assume that the borrower's utility depends on an amount of equity in his house. h_t is the current local house price, which is calculated from h_0 and Case-Shiller house price index at a zipcode-level; bal_t is remaining balance as of period t, which includes the total amount of previously unpaid required monthly payments. Then $h_t - bal_t$ is an amount of equity in the house in period t. We assume that the borrower receives a different utility from his equity, depending on a loan status. The reason is because once the borrower becomes delinquent for at least three months, a foreclosure process will start. During a foreclosure process, the borrower will evaluate his equity in the house differently from when he is current. For one thing, in states with deficiency judgment laws, lenders apply unpaid debt (negative home equity) to the borrower's other asset which the borrower may value more or less than his home equity.

The borrower also receives (dis)utility from making a positive amount of payments. α_s is a coefficient for payment amounts (a_{bt}) that depends on the borrower's type s, and v_t is a preference shock to the "price" coefficient α_s. As will be come clearer later, we make the type distribution depend on the borrower's initial characteristics x_0 such as income, credit (FICO) score and loan-to-value ratio (LTV) at the time of loan origination. Therefore the characteristics will eventually affect the borrower's payment behavior. For the preference shock v_t, we assume that $v_t \sim logN(\beta_{unr} unr_t, \lambda_v)$, where β_{unr} and λ_v are parameters, and that v_t is i.i.d across time and individuals conditional on county-level unemployment rates unr_t. With this specification, changes in local unemployment rates affect the borrower's behavior by shifting the distribution for v_t.

Lastly, $\epsilon_t(a_{bt})$ is another kind of preference shock that is additive and separable. We assume that $\epsilon_t(a_{bt}) = \epsilon_t(a'_{bt}) \equiv \epsilon_{1t}$ for any $a_{bt}, a'_{bt} > 0$, and we denote that $\epsilon_{0t} \equiv \epsilon_t(0)$. Moreover, we assume that ϵ_{jt} is i.i.d with type 1 extreme value distribution across individuals, time, and $j = 0, 1$.[13]

[11] Ideally we would like to include characteristics of a house in the consumption value. Unfortunately, such information is not available. We use the initial house price at the origination as a proxy for time-invariant house characteristics.

[12] As we discuss later, we will normalize utility from a foreclosure to zero. Therefore the consumption value of living in a house is relative to the utility from a foreclosure.

[13] Note that the borrower receives the same shock for any choices with which he makes a positive amount of payments. We do not allow for an additive and separable choice-specific shock for all choices because such a model could lead to a perverse incentive for the borrower to miss a payment. In the model, the borrower's choice set changes endogenously, and the number of altenatives in a choice set increases as the borrower

3.1.2 The Lender

Choice Set The lender takes an action after the borrower has made his payment decision and only when the borrower does not make payment in period t and becomes at least two-month-delinquent. More specifically, let δ_t' denote the borrower's delinquent status after the borrower has made his payment decision in period t. We then have:

$$\delta_t' = \begin{cases} 0, & \text{if } a_{bt} = (\delta_t + 1)mp; \\ 1, & \text{if } a_{bt} = \delta_t mp; \\ 2, & \text{if } a_{bt} = (\delta_t - 1)mp \text{ and } \delta_t \geq 1; \\ \delta_t + 1, & \text{if } a_{bt} = 0 \text{ and } \delta_t \geq 2. \end{cases}$$

The lender's choice set is defined as follows:

$$A_l(\delta_t') = \begin{cases} \{wait, fc, r_{mod}\} & \text{if } \delta_t' \geq 2 \text{ and } a_{bt} = 0; \\ \{wait\} & \text{otherwise} \end{cases}$$

If the lender chooses $wait$, then he does not make any changes to the mortgage and leave it as it is. The term fc stands for foreclosure, which does not mean the initiation of a foreclosure process but an actual liquidation of a house.[14] Lastly, r_{mod} means that the lender modifies the loan and decreases interest rates by r_{mod} percentage points. Note that we restrict the lender to interest rate modification and if the new rate is the same as the old interest rate, then the mortgage modification is equivalent to recapitalization. In reality, there are other mortgage modification tools such as principal reduction. However, according to our data less than 2% of modified loans received principal reduction, and other forms of a modification are even rarer.

Utility After observing the borrower's action a_{it}, the lender in state x_t and preference shock $\omega_t \sim W$ receives the payoff of $u_l(a_{lt}; x_t, a_{bt}, \omega_t)$ from action $a_{lt} \in A_l(\delta_t')$:

$$u_l(a_{lt}; x_t, a_{bt}, \omega_t) = a_{bt} + c(x_t, a_{lt}; \theta) + \lambda_\omega \omega_t(a_{lt}).$$

The lender's per-period payoff has three components: (i) an amount of payments he receives from the borrower in the current period (a_{bt}); (ii) net costs associated with an action taken

misses a payment in a previous month. With a choice-specific shock for each alternative, a larger choice set will result in a larger expected utility, which will give the borrower an incentive to miss a payment.

[14] We choose not to model the lender's decision to initiate a mortgage explicitly because the borrower can still live in a house until his house is sold by the lender. Moreover, some borrowers come back to be current on their mortages when they are in a foreclosure process, and many loans are modified during the process. In this sense, being in a foreclosure process is not very different from being serously delinquent.

by the lender (c); (iii) choice-specific preference shocks associated with each of the lender's actions (ω). The first term is obvious. For the second term, we discuss its specification for each a_{lt}:

Foreclosure If the lender forecloses a house, then the game between the borrower and lender ends and each party receives a terminal payoff.[15] Remember that we defined a foreclosure in the model as an actual liquidation of a house, not the initiation of a foreclosure process. For the lender's period utility from a foreclosure, we assume that

$$c(x_t, fc; \theta) = \theta_{fc} + \theta_h h_t + \theta_{bal} bal_t + \theta_{state}.$$

The first term is a constant for a foreclosure. The second term represents payoffs from a foreclosure related to the current house price. It is obvious that the payoffs will depend on the current condition of the housing market. We also let the payoffs depend on the remaining balance of a mortgage (bal_t). Lastly, the payoffs also depend on a state in which a house is located. This is because different states have different laws regarding a foreclosure.

Modification If the lender chooses to modify a loan, then he reduces an interest rate by r_{mod} ($\in R$) percentage points. Let us denote the original interest rate as r_0. If the lender chooses r_{mod} in period t, then a new amount of monthly required payments is calculated as if the lender issued a new loan with the initial balance of bal_t, a new interest rate of $r_0 - r_{mod}$ and remaining periods of $T - t$. Moreover, a modification changes a loan status to being current in the following period. In other words, $\delta_{b,t+1} = 0$ after a modification in period t. We assume that the change in a contract between the borrower and lender due to modification in period t will be effective from period $t + 1$. At the time of a modification, we assume that $c(x_t, r_{mod}; \theta) = \theta_{mod}$ where θ_{mod} is a fixed cost of modification, which will capture any costs associated with changing a loan contract that does not depend on a new interest rate. Benefits or costs to the lender which depend on r_{mod} will be realized through his continuation payoffs.

In an actual implementation, we assume $R = \{0, 1.5, 3, 4.5\}$ for tractability. That is, the lender can reduce an interest rate by 0, 1.5, 3, or 4.5 percentage points.[16] The difference between waiting and modification with no interest reduction is that the latter changes (i) loan status to being current and (ii) an amount of monthly payments. Note that an amount of monthly payments changes even with a modification with zero interest reduction because bal_t includes all of the previously unpaid monthly payments. As a result, monthly payments must increase with a modification without a reduction of interest rates.

[15] Terminal payoffs from a foreclosure for the borrower and lender will be specified later in this section.

[16] As discussed in the data section, the first option is the most common form of modification.

Waiting For the option of waiting, we normalize $c(x_t, wait; \theta) = 0$ for any x_t and parameter θ.

For the preference shock, we assume that $\omega(a_{lt})$ is distributed as nested logit, which is independent across lenders, time and nests. We allow for three nests: waiting, foreclosure and modification. The first two nests are trivial and have only one action in each of them. The nest for modification contain $a_{lt} = r_{mod} \in R$. This functional form assumption for $\omega(a_{lt})$ was chosen in order to allow for correlation between shocks for different options for modification, which seems very likely. Formally, let $\omega \equiv \{\omega(a_{lt})\}_{a_{lt} \in A_{lt}(\delta'_t)}$ and $\omega \sim W(\lambda_{mod})$ where W is a nested logit distribution with parameter λ_{mod} for the nest for modification.[17] We normalize the mean and standard deviation of the distribution W. Instead, we estimate λ_ω for the standard deviation and constant terms for the mean.

3.1.3 Terminal Payoffs

There are two ways that the game between the borrower and lender ends. First, the lender forecloses a seriously delinquent borrower's house. Second, a game reaches the final period T.

Foreclosure In case that the lender forecloses a house, the game between the borrower and lender ends. We normalize continuation payoffs from a foreclosure in period t to zero for both players: for $i \in \{b, l\}$ and any possible x_{t+1},

$$V_i(x_{t+1}, fc) = 0.$$

This means that a player's utility is measured relatively to the payoffs he receives from a foreclosure.

Final Period In reality, the final period is equal to a loan maturity unless the borrower refinances and sells a house. A typical mortgage has a maturity of 30 years or 360 months. Apparently, solving a model with $T = 360$ is very demanding computationally. Moreover, an interaction between the borrower and lender of our interest occurs in a relatively early stage of a mortgage loan life cycle. For this reason, we assume that $T = 60$. In other words, we model behaviors of the borrower and lender for first five years of a mortgage life cycle. We assume that after the borrower and lender makes a decision in period T, they receives a final payoff, which is a parametric function of state variables at the time. For the borrower

[17] Since the nests for waiting and foreclosure have only one alternative in each of them, it is not necessary to specificy a parameter for the two nests.

and lender in period T, their continuation payoff is given by: for $i \in \{b, l\}$,

$$V_i(x_{T+1}) = \gamma_{i0} + \gamma_{i,bal} bal_t + \gamma_{i,mp} mp_{T+1}$$

That is, the final payoff depends on remaining balance in a mortgage, which includes an amount in arrears, and an amount of monthly payments.

3.1.4 Loan Status

In an actual implementation, we make assumptions on loan status for tractability. We assume that $\delta_{bt} \leq 14$ and that if $\delta_{bt} > 14$, then the lender forecloses the house with probability one. In reality, this is not necessary true, but a modification is rarely observed for a borrower with $\delta_{bt} > 14$. Moreover, we assume that $A_b(\delta_{bt}) = \{0\}$ for $\delta_{bt} > 6$. This means that if the borrower misses the payment for more than 6 months, then the borrower's only alternative is not to pay. since about 99% of borrowers with $\delta_{bt} > 6$ do not pay their monthly payment in the data, we do not find this assumption very restrictive. With this assumption, a borrower with $6 < \delta_{bt} \leq 14$ does not actively make a decision but wait for possible modifications by the lender. Should the lender modifies a loan for such a borrower, the borrower starts to make an active decision again in the subsequent periods.

3.2 Transition between States

As discussed above, loan status evolves over time, and its transition depends on actions taken by the borrower and lender. Other than loan status, there are two other states which evolve over time: local house prices (h_t) and local unemployment rates (unr_t). Unlike loan status, the transition of the two states do not depend on actions of the borrower and lender. Since we calculate h_t with the initial house price at the time of loan origination (h_0) and local house price index (hpi_t), we just need to consider the transition of hpi_t for the transition of h_t.

In the actual implementation, we discretize hpi_t and unr_t and assume that they follow the first-order Markov process. For hpi_t, we allow for five discrete points for a deviation of hpi_t from $E_t[hpi_t]$, which is local mean house price index. For unr_t, we allow for six discrete points for a deviation of unr_t from $E_t[unr_t]$, which is also local mean unemployment rates.

3.3 Type Distribution

Since we do not observe a borrower's permanent unobserved heterogeneity $s \in S$ by assumption, we have to specify a distribution for the type. Let $\pi_s(x_0)$ denote the probability for the borrower with initial characteristics x_0 to be of type s. We thus have $\sum_{s \in S} \pi_s(x_0) = 1$

for all $x_0 \in X$. In an actual implementation, we assume that there are two types so that $S = \{1, 2\}$, and

$$\pi_2(x_0) = \frac{\exp(x_0\tau)}{1 + \exp(x_0\tau)},$$

where τ is a parameter we estimate.

3.4 Information Structure

We assume that each player observes x_t, which includes loan characteristics, the borrower's characteristics (except type s) and local house prices and unemployment rates. Moreover, each player perfectly observes all of actions the other player has chosen. In the model, there exists information asymmetry regarding the borrower's type and each player's preference shock. The borrower's permanent type s is his own private information. Each player's preference shock in each period (ξ_t and ω_t for the borrower and lender, respectively) is his own private information. Because of this private information, a player expects that the other player chooses a certain action with some probability, not in a deterministic way.

3.5 Strategies

A player i's strategy σ_i is a complete description of actions that will be taken in each state $(x_t, s, \xi_t, \omega_t)$ for $t = 1, \cdots, T$. Let us define σ_{it} to be player i's strategy for period t so that $\sigma_i = (\sigma_{i1}, \cdots, \sigma_{iT})$. Since we assume that each player's preference shock in each period is private information, ω_t will not affect the borrower's strategy. Moreover, the borrower moves before observing the lender's action in the same period. Therefore the borrower's strategy for period t will depend on observed states, his type, and his preference shock. That is, $\sigma_{lt}(x_t, s, \xi) \in A_b(\delta_t)$.

The lender's strategy will depend on different objects because of different information structure and timing of moves. In the model, the lender does not know the borrower's unobserved type s or preference shock ξ. Moreover, the lender moves after observing the borrower's action in the same period. Thus the lender's strategy for period t will depend on observed state, the borrower's action in period t, and his preference shock: $\sigma_{bt}(x_t, a_{bt}, \omega_t) \in A_l(\delta_t')$.

3.6 Value Functions

Let $\beta \in (0, 1)$ denote the discount rate, and let Γ_i be a set of strategies for player i so that $\sigma_i \in \Gamma_i$. Moreover, let $F(x_{t+1}|x_t, a_{bt}, a_{lt})$ be the distribution of x_{t+1} conditional on x_t and each player's action a_{bt} and a_{lt}. Given the other player's strategy, the borrower and the lender chooses σ_b and σ_l in order to maximize the expected discounted sum of utility in each period t. In writing the borrower's value function, we will use the lender's conditional choice probabilities $p_l = (p_{l1}, \cdots p_{lT})$ where

$$p_{lt}(a_{lt}; x_t, a_{bt}) \equiv \int_{\omega_t} \mathbf{1}[\sigma_l(x_t, a_{bt}, \omega_t) = a_{lt}] dW(\omega_t).$$

Since ω is private information of the lender, p_l is how the borrower thinks the lender will behave in the future according to the lender's strategy σ_l. The borrower's value function in period t given the lender's strategy σ_l,

$$V_b(x_t, s, \xi_t; p_l) = \max_{a \in A_b(\delta_t)} u_b(a; x_t, s, \xi_t) + \beta \sum_{k \in A_l(\delta_t')} \left(\int_{x', \xi'} V_b(x', s, \xi'; p_l) dF(x'|x_t, a, k) dG(\xi') \right) p_{lt}(k; x_t, a)$$

In the value function, the borrower considers utility for the current period and the effects of his action on the lender's action, which in turn will affect the transition from x_t to x_{t+1}.

For the lender's value function, let us first define the borrower's conditional choice probability $p_b = (p_{b1}, \cdots, p_{bT})$:

$$p_{bt}(a_{bt}; x_t) = \sum_{s \in S} \pi(s|x_0) \int_{\epsilon_t} \mathbf{1}[\sigma_b(x_t, s, \epsilon_t) = a_{bt}] dG(\epsilon_t).$$

The lender's value function, given the borrower's action in the same period a_{bt} and choice probability p_b, can be written as:

$$V_l(x_t, a_{bt}, \omega_t; p_b) = \max_{k \in A_l(\delta_t')} u_l(k; x_t, a_{bt}, \omega_t) + \beta \int_{x', \omega'} \left(\sum_{a' \in A_b(\delta_{b,t+1})} V_l(x', a', \omega'; p_b) p_{b,t+1}(a'; x_{t+1}) \right) dG(\omega') dF(x_{t+1}|x_t, a_{bt}, k)$$

In the lender's value function, the lender considers utility for the current period and the effects of his action on transition from x_t to x_{t+1}, which in turn will affect the borrower decision in the next period before the lender's move.

3.7 Equilibrium

Our equilibrium concept for the model is a Markov-perfect equilibrium. The key assumption in the concept is that players' strategies are functions of only payoff-relevant state variables. In a Markov-perfect equilibrium, a player's strategy is a mapping of his state to an action and is a best response to the other player's strategy.

4 Estimation

One possible way to estimate the model is maximum likelihood, which requires numerically solving an equilibrium of the model at each iteration. The procedure is not only time-consuming but also requires an econometrician to know which equilibrium is played should there exist multiple equilibria. Indeed, it is not clear a priori whether there exists a unique equilibrium in the model due to asymmetric information between the borrower and lender.

For this reason, we estimate the model with a multi-step estimation procedure which makes use of the fact that a player's equilibrium strategy is the best response to the other player's equilibrium strategy. By the definition of a Market-perfect equilibrium, a player's equilibrium strategy σ_i^* must maximize his expected payoffs given the other player's strategy σ_{-i}^*. The property of an equilibrium implies that as long as σ_{-i}^* is known to an econometrician, we can treat a player i's problem as a single-agent dynamic maximization problem. Then we can apply an estimation method available for a single-agent dynamic maximization problem to estimate parameters for player i. Once we estimate the parameters, we come back to the other player's problem and estimate the rest of the parameters.

A critical part of our estimation procedure is that an econometrician must know an equilibrium strategy played by at least one player. By inspecting equation (**??**), note that the borrower's value function depends on the lender's strategy σ_l only through the lender's conditional choice probability p_l, which depend only on observed states and the borrower's action (x_t, a_{bt}). Although we do not directly observe p_l, we can consistently estimate p_l using the lender's observed choices in different states (x_t, a_{bt}).[18] Once we have a consistent estimate of p_l, we can estimate parameters that govern the borrower's behaviors using a standard estimation method for a single-agent dynamic model. In the end, we come back to the lender's problem and estimate the rest of the parameters in the model.

We now discuss our estimation procedure in more details. We estimate the model with the following three-step estimation procedure:

4.1 The First Step

In the first step, we estimate the lender's conditional choice probabilities p_l directly from the data. At this stage, we do not need to estimate structural parameters that govern the lender's behaviors because all we need to know to solve the borrower's problem is how the lender will behave in each state. With dataset with a very large number of observations, it would be possible even to estimate p_l nonparametrically. Due to a small sample, however,

[18] This part of our estimation procedure is very similar to other estimation methods of dynamic models that use conditional choice probabilities such as Hotz and Miller (1993), Hotz, Miller, Sanders and Smith (1994) ,Aguirregabiria and Mira (2007), Bajari, Benkard and Levin (2007), Pakes, Ostrovsky and Berry (2007), and Pesendorfer and Schmidt-Dengler (2008)

we use a parametric model to estimate p_l like other papers that use a similar method. Since we assume that a choice-specific preference shock is distributed as nested logit in the model, we use a nested logit model to estimate p_l as a function of observed states, their polynomials and interactions.

4.2 The Second Step

Given consistent estimates of the lender's choice probabilities $\hat{p}_l$, we estimate parameters for the borrower in the second step as if we estimate a single-agent problem of the borrower. We estimate the parameters for the borrower using simulated maximum likelihood.[19] This estimation method requires calculating the value function in every possible state that can be reached with a positive probability. The model does not have an analytical solution, but the finite horizon dynamic programming model can be numerically solved by backward recursion, starting from the final period T. Since the number of possible states is very large, it is computationally challenging to solve for the exact value function for each possible state. Therefore, we adopt the approximation method developed in Keane and Wolpin (1994) in which the value functions are evaluated at a random subset of the states, and the values are used for interpolation at non-evaluated states.

4.3 The Third Step

Once we estimate parameters for borrowers, we estimate parameters for lenders. Using the estimated parameters for borrowers, we can calculate the borrower's conditional choice probabilities with the model. Since the lender does not have unobserved heterogeneity, we use an estimator based on conditional choice probabilities in Hotz and Miller (1993).

5 Results

We have estimated the first and second step of the estimation procedure. We plan to complete the third step soon.

[19] The fact that there is a preference shock (v_t) to enter in a multiplicative way prevents us from applying an estimation method based on conditional choice probabilities. And the lack of additive and separable choice-specific shock for the borrower's every choice is another reason. If the model allows for an additive and separable preference shock for each option, and if we remove v_t from the model, then such a method will be applicable. However, such a model will have the critical drawback as discussed above in the model section.

5.1 Parameter Estimates

The estimates of parameters for the borrower are reported in Table 2.[20] The estimates show that a higher monthly payment reduces the probability of making payments for all consumers and that a borrower of type 2 will have a higher disutility from payments. The parameter estimates for the initial type probability (τ) indicate that those with a lower initial LTV, a higher income, a higher FICO score, and full documentation at the origination of their mortgages are more likely to be of type 1, who will be more likely to make payments. The coefficient for local unemployment rates (β_{unr}) implies that a borrower living in a region with a higher local unemployment rate will be more likely to receive a negative shock to his disutility from payments. Moreover, the estimates of $\beta_{eq,1}$ and $\beta_{eq,2}$ show that a borrower feels differently towards their home equity. If a borrower is delinquent for at least 4 months, then a borrower with a smaller amount of equity receive less disutility from his delinquent status. Since we do not recognize the initiation of a foreclosure process as a separate state, and since those who are seriously delinquent are very likely in a foreclosure status, the estimate for $\beta_{eq,2}$ seems to capture a borrower's utility from his equity in the house in a foreclosure process

5.2 Model Fit

We report model fit for the data on borrowers in Table (3) to (4). The model is able to fit important patterns in the data on a borrower's payment behavior despite a room for an improvement. The results in Table (3) show that the model is able to generate a pattern in the data that those who have already been late in payments are more likely to miss a payment again due to selection. However, the model over-predicts the payment behavior for those who are seriously delinquent (at least 2 month late). The results in Table (4) show that the model can match a borrower's payment behavior depending on aggregate variables such as local house prices and unemployment rates. Model fit regarding the relationship between a borrower's behavior and his characteristics is presented in Table (5). Although the model is able to fit an overall pattern of the data, the results show that there is still a room for improvement.

5.3 Counterfactual Simulations

Using the estimated model for borrowers, we ask how a borrower's payment behavior changes, depending on his expectation of possible modification in the future. To answer the question, we exogenously change a lender's probability of modification and simulate a borrower's optimal response to the exogenous change. Through this counterfactual exercise, we quantify

[20] Standard errors have not been calculated yet.

a borrower's strategic default, which is considered one of important factors making lenders reluctant to offering a modification.

Table (6) presents the simulation results. The baseline results are from the estimated model. For the results in the column for 'high prob', we increase a lender's probability of offering a modification. Recall that there are four different kinds of modification available in the model. For this simulation, we increase the probability of each kind of modification by 10 percentage points in each period should a borrower misses a payment. At the same time, we do not change the probability of a foreclosure but adjust the probability of waiting so that all of the probabilities sum up to one.[21] For the results in the column for 'low prob', we simply set the probability of a modification to zero, fix the probability of a foreclosure, and adjust the probability of waiting so that all of the probabilities sum up to one.

The results show that a borrower's expectation about a lender's behavior has a large impact on a borrower's payment behavior. In the case of 'high prob', a borrower, who is current or delinquent for one or two months, is less likely to make a payment, compared to their predicted behavior in the baseline. For an increase in the probability of each kind of modification up to 10 percentage points, a borrower's probability of making payments decrease by 13-18% depending on loan status. In the case of 'low prob', where a borrower does not expect any modification at all, a borrower is now more likely to make a payment than the baseline, and the probability of making payments increase by about 5.7% and 7.5% for a borrower who are delinquent for one and two months, respectively. Given that the probability that at least one of four kinds of modification is offered by a lender in a given period is very low (usually around 1-2%), we view the changes in a borrower's behavior non-negligible.

The results highlight a potentially important reason why a lender is reluctant to offering a modification to a delinquent borrower. Given the significant response from a borrower to changes in a lender's strategy, a lender would fear that a generous strategy of offering a modification more frequently will make an otherwise current borrower default on his mortgage in hopes of lowering monthly payments through a modification. Moreover, this problem will be more serious in a setting where a lender cannot perfectly distinguish different types of borrowers with different abilities to make payments, which is the setting we model.

[21] In case that the new probability of a modification becomes so high that the probability of waiting has to be negative, we increase the probability of a modification up to a point where the probability of waiting has to be zero.

A Appendix

A.1 Parameter Estimates

Table 2: Estimation Results

	Parameters	Estimates
Utility	β_0	-3.481
	β_h	.0380
	β_{pay}	3.060
	β_{crnt}	.413
	$\beta_{eq,1}$	0.005
	$\beta_{eq,2}$	-0.440
	α_1	-0.970
	α_2	-6.355
	α_{sq}	-0.142
	β_{unr}	0.154
	λ_v	1.454
Type	τ_0	10.829
	τ_{Time_0}	-0.315
	τ_{LTV_0}	1.620
	τ_{income_0}	-0.297
	τ_{FICO}	-1.529
	$\tau_{fulldoc}$	-0.607

A.2 Model Fit

Table 3: Loan Status and Probability of Making a Payment

Loan Status	Data	Model
Current	0.912	0.888
1-month-delinquent	0.711	0.729
2-month-delinquent	0.555	0.649
3- month-delinquent	0.093	0.232
$\geq$ 4-month-delinquent	0.044	0.073

Table 4: Aggregate Stochastic States and Probability of Making a Payment

Unemp Rates	Data	Model
$unr_t \leq 25\%$	0.896	0.884
$25\% < unr_t \leq 50\ \%$	0.888	0.861
$50\% < unr_t \leq 75\ \%$	0.824	0.806
$75\% < unr_t$	0.734	0.744
House Prices	**Data**	**Model**
$h_t \leq 25\%$	0.796	0.810
$25\% < h_t \leq 50\ \%$	0.806	0.802
$50\% < h_t \leq 75\ \%$	0.841	0.812
$75\% < h_t$	0.897	0.871

For example, that $unr_t < 25\%$ means that a current local unemployment rate is not greater than the 25th-percentile of the distribution of local unemployment rates. The numbers for this row are calculated with information on borrowers living in a region whose local unemployment rates are not greater than the 25%-percentile of the local unemployment distribution.

Table 5: Borrower Characteristics and Probability of Making a Payment

Monthly Payments	Data	Model
Below Median	0.828	0.832
Above Median	0.843	0.816
LTV at the origination	**Data**	**Model**
Below Median	0.845	0.845
Above Median	0.825	0.812
Income at the origination	**Data**	**Model**
Below Median	0.812	0.815
Above Median	0.854	0.834
FICO	**Data**	**Model**
Below Median	0.785	0.793
Above Median	0.861	0.846
Full Documentation	**Data**	**Model**
No	0.841	0.820
Yes	0.831	0.828

A.3 Counterfactual Simulation

Table 6: Probability of Modification and a Borrower's Behavior

Loan Status	Baseline	High Prob	Low Prob
Current	0.888	0.759	0.890
1-month-delinquent	0.729	0.580	0.767
2-month-delinquent	0.648	0.537	0.694

- High Prob: for this simulation, we increase the probability of each kind of modification by 10 percentage points in each period should a borrower misses a payment. At the same time, we fix the probability of a foreclosure and adjust the probability of waiting so that all of the probabilities sum up to one. In case that the new probability of a modification becomes so high that the probability of waiting has to be negative, we increase the probability of a modification up to a point where the probability of waiting has to be zero.

- Low Prob: we set the probability of a modification to zero, fix the probability of a foreclosure, and adjust the probability of waiting so that all of the probabilities sum up to one.