Technical Appendix for:

Macroeconomic Interdependence and the International Role of the Dollar.*

Linda Goldberg
Federal Reserve Bank of New York and NBER

Cedric Tille
Geneva Graduate Institute of International and Development Studies and CEPR

January 22, 2008

*Linda Goldberg: linda.goldberg@ny.frb.org, Cedric Tille: cedric.tille@graduateinstitute.ch. The views expressed in the paper are those of the authors and do not necessarily represent those of the Federal Reserve Bank of New York or the Federal Reserve System.

1 A simple center-periphery model

1.1 Intratemporal allocation of consumption

1.1.1 Geographical structure

The world is made of three countries: A, B and C. Country A represents a "center" country, while countries B and C are "periphery" countries. The size of the world economy is set to unity. Country A accounts for half the world, while each of the periphery countries accounts for one-quarter of the world. Each country is inhabited by a representative consumer.

Consumers purchase a continuum of differentiated brands, that are indexed along a unit interval. Firms in country A produce brands on the $0-0.5$ interval, firms in country B produce brands on the $0.5-0.75$ interval, and firms in country C produce brands on the $0.75-1$ interval. All goods are traded, and we allow for home bias in consumption between the center and periphery goods.

1.1.2 Notation

Consumptions levels are indexed with a subscript for the country where consumption takes place, and a superscript for the country where the good is produced. Specifically, $C_i^j(z)$ is the consumption in country i of the brand z produced in country j. Individual brands are aggregated into indexes, as detailed below, and C_i^j is the consumption in country i of the index of all brands produced in country j. The indexes themselves are aggregated further into the overall consumption, with C_i being the overall consumption index in country i.

The prices of the various goods are indexes along similar lines. $P_i^j(z)$ is the price paid for the consumers in country i for each unit the brand z produced in country j. The prices of the various brands produced in a given country are aggregated into a country-of-origin price index, with P_i^j being the price index charged in country i for the brands produced in country j. These indexes are in turn aggregated in the overall consumer price index P_i. Prices are expressed in the currency of the country where the goods are consumed, namely i.

1.1.3 Country A

The representative consumer in the center country A allocates her consumption across the various brands to maximize the following index:

$$C_A = (\alpha)^{-\alpha}\left(\frac{1-\alpha}{2}\right)^{-(1-\alpha)}\left(C_A^A\right)^{\alpha}\left(C_A^B C_A^C\right)^{\frac{1-\alpha}{2}}$$

where:

$$\begin{aligned}
C_A^A &= \left[(2)^{\frac{1}{\lambda}}\int_0^{0.5}\left(C_A^A(z)\right)^{\frac{\lambda-1}{\lambda}}dz\right]^{\frac{\lambda}{\lambda-1}} \\
C_A^B &= \left[(4)^{\frac{1}{\lambda}}\int_{0.5}^{0.75}\left(C_A^B(z)\right)^{\frac{\lambda-1}{\lambda}}dz\right]^{\frac{\lambda}{\lambda-1}} \\
C_A^C &= \left[(4)^{\frac{1}{\lambda}}\int_{0.75}^{1}\left(C_A^C(z)\right)^{\frac{\lambda-1}{\lambda}}dz\right]^{\frac{\lambda}{\lambda-1}}
\end{aligned}$$

$\lambda > 1$ is the elasticity of substitution between produced in the same country. The elasticity of substitution between goods produced in different countries is set at 1. $\alpha \in [0.5, 1]$ is the degree of home bias, in terms of periphery vs. center goods. If $\alpha = 0.5$ there is no home bias and the structure of the consumption basket is the same for all agents worldwide. If $\alpha = 1$ the center and the periphery are disconnected, and the consumer in country A consumes only local goods. There is no bias between the various periphery goods.

The allocation of consumption reflects relative prices:

$$\begin{aligned}
C_A^A &= \alpha\frac{P_A}{P_A^A}C_A \quad ; \quad C_A^B = \frac{1-\alpha}{2}\frac{P_A}{P_A^B}C_A \quad ; \quad C_A^C = \frac{1-\alpha}{2}\frac{P_A}{P_A^C}C_A \\
C_A^A(z) &= 2\left[\frac{P_A^A(z)}{P_A^A}\right]^{-\lambda}C_A^A \quad ; \quad C_A^B(z) = 4\left[\frac{P_A^B(z)}{P_A^B}\right]^{-\lambda}C_A^B \\
C_A^C(z) &= 4\left[\frac{P_A^C(z)}{P_A^C}\right]^{-\lambda}C_A^C
\end{aligned}$$

We derive the usual cost-minimizing price indexes as:

$$\begin{aligned} P_A &= \left(P_A^A\right)^{\alpha}\left(P_A^B P_A^C\right)^{\frac{1-\alpha}{2}} \qquad (1)\\ P_A^A &= \left[2\int_0^{0.5}\left[P_A^A(z)\right]^{1-\lambda}dz\right]^{\frac{1}{1-\lambda}} \\ P_A^B &= \left[4\int_{0.5}^{0.75}\left[P_A^B(z)\right]^{1-\lambda}dz\right]^{\frac{1}{1-\lambda}} \quad ; \quad P_A^C = \left[4\int_{0.75}^{1}\left[P_A^C(z)\right]^{1-\lambda}dz\right]^{\frac{1}{1-\lambda}} \end{aligned}$$

1.1.4 Country B

The representative consumer in the center country B allocates her consumption across the various brands to maximize the following index:

$$C_B = (1-\alpha)^{-(1-\alpha)}\left(\frac{\alpha}{2}\right)^{-\alpha}\left(C_B^A\right)^{1-\alpha}\left(C_B^B C_B^C\right)^{\frac{\alpha}{2}}$$

where:

$$\begin{aligned} C_B^A &= \left[(2)^{\frac{1}{\lambda}}\int_0^{0.5}\left(C_B^A(z)\right)^{\frac{\lambda-1}{\lambda}}dz\right]^{\frac{\lambda}{\lambda-1}} \\ C_B^B &= \left[(4)^{\frac{1}{\lambda}}\int_{0.5}^{0.75}\left(C_B^B(z)\right)^{\frac{\lambda-1}{\lambda}}dz\right]^{\frac{\lambda}{\lambda-1}} \\ C_B^C &= \left[(4)^{\frac{1}{\lambda}}\int_{0.75}^{1}\left(C_B^C(z)\right)^{\frac{\lambda-1}{\lambda}}dz\right]^{\frac{\lambda}{\lambda-1}} \end{aligned}$$

The allocation of consumption is:

$$\begin{aligned} C_B^A &= (1-\alpha)\frac{P_B}{P_B^A}C_B \quad ; \quad C_B^B = \frac{\alpha}{2}\frac{P_B}{P_B^B}C_B \quad ; \quad C_B^C = \frac{\alpha}{2}\frac{P_B}{P_B^C}C_B \\ C_B^A(z) &= 2\left[\frac{P_B^A(z)}{P_B^A}\right]^{-\lambda}C_B^A \quad ; \quad C_B^B(z) = 4\left[\frac{P_B^B(z)}{P_B^B}\right]^{-\lambda}C_B^B \\ C_B^C(z) &= 4\left[\frac{P_B^C(z)}{P_B^C}\right]^{-\lambda}C_B^C \end{aligned}$$

where the price indexes are:

$$
\begin{aligned}
P_B &= \left(P_B^A\right)^{1-\alpha}\left(P_B^B P_B^C\right)^{\frac{\alpha}{2}} \qquad (2)\\
P_B^A &= \left[2\int_0^{0.5}\left[P_B^A(z)\right]^{1-\lambda}dz\right]^{\frac{1}{1-\lambda}}\\
P_B^B &= \left[4\int_{0.5}^{0.75}\left[P_B^B(z)\right]^{1-\lambda}dz\right]^{\frac{1}{1-\lambda}} \quad ; \quad P_B^C = \left[4\int_{0.75}^{1}\left[P_B^C(z)\right]^{1-\lambda}dz\right]^{\frac{1}{1-\lambda}}
\end{aligned}
$$

1.1.5 Country C

The representative consumer in the center country C allocates her consumption across the various brands to maximize the following index:

$$C_C = (1-\alpha)^{-(1-\alpha)}\left(\frac{\alpha}{2}\right)^{-\alpha}\left(C_C^A\right)^{1-\alpha}\left(C_C^B C_C^C\right)^{\frac{\alpha}{2}}$$

where:

$$
\begin{aligned}
C_C^A &= \left[(2)^{\frac{1}{\lambda}}\int_0^{0.5}\left(C_C^A(z)\right)^{\frac{\lambda-1}{\lambda}}dz\right]^{\frac{\lambda}{\lambda-1}}\\
C_C^B &= \left[(4)^{\frac{1}{\lambda}}\int_{0.5}^{0.75}\left(C_C^B(z)\right)^{\frac{\lambda-1}{\lambda}}dz\right]^{\frac{\lambda}{\lambda-1}}\\
C_C^C &= \left[(4)^{\frac{1}{\lambda}}\int_{0.75}^{1}\left(C_C^C(z)\right)^{\frac{\lambda-1}{\lambda}}dz\right]^{\frac{\lambda}{\lambda-1}}
\end{aligned}
$$

The allocation of consumption is:

$$
\begin{aligned}
C_C^A &= (1-\alpha)\frac{P_C}{P_C^A}C_C \quad ; \quad C_C^B = \frac{\alpha}{2}\frac{P_C}{P_C^B}C_C \quad ; \quad C_C^C = \frac{\alpha}{2}\frac{P_C}{P_C^C}C_C\\
C_C^A(z) &= 2\left[\frac{P_C^A(z)}{P_C^A}\right]^{-\lambda}C_C^A \quad ; \quad C_C^B(z) = 4\left[\frac{P_C^B(z)}{P_C^B}\right]^{-\lambda}C_C^B\\
C_C^C(z) &= 4\left[\frac{P_C^C(z)}{P_C^C}\right]^{-\lambda}C_C^C
\end{aligned}
$$

where the price indexes are:

$$\begin{aligned} P_C &= \left(P_C^A\right)^{1-\alpha}\left(P_C^B P_C^C\right)^{\frac{\alpha}{2}} \qquad (3) \\ P_C^A &= \left[2\int_0^{0.5}\left[P_C^A(z)\right]^{1-\lambda}dz\right]^{\frac{1}{1-\lambda}} \\ P_C^B &= \left[4\int_{0.5}^{0.75}\left[P_C^B(z)\right]^{1-\lambda}dz\right]^{\frac{1}{1-\lambda}} \quad ; \quad P_C^C = \left[4\int_{0.75}^{1}\left[P_C^C(z)\right]^{1-\lambda}dz\right]^{\frac{1}{1-\lambda}} \end{aligned}$$

1.2 Intertemporal allocation

We consider a one-period model. Firms set their prices at the beginning of the period. Shocks then occur, the monetary authorities react to these shocks, and consumption and production take place. While firms set prices before the realization of shocks, they do so knowing the distribution of shocks and the rules followed by the monetary authorities. Once shocks are realized, firms meet the demand they face at their posted prices.

The consumer in country i maximizes a simple utility over consumption, real balances and hours worked:

$$U_i = E\left[\ln(C_i) + \chi\ln\left(\frac{M_i}{P_i}\right) - \kappa H_i\right]$$

where E denotes the expectation operator. C_i is the aggregate consumption index, M_i/P_i denotes the real money balances and H_i denotes the hours worked by the consumer. χ and κ are scaling parameters. The budget constraint for the consumer in country i is:

$$P_iC_i + M_i = \Pi_i + W_iH_i - T_i \qquad (4)$$

where Π_i denotes the profits of the firms in country i, which are owned by the consumer, W_i the wage rate and T_i a lump-sum tax paid to the government of country A.[1] The first-order conditions with respect to real balances and hours worked are:

$$M_i = \chi P_iC_i \qquad W_i = \kappa P_iC_i = \frac{\kappa}{\chi}M_i \qquad (5)$$

[1] Without loss of generality we assume that initial cash holdings are zero.

1.3 Structure of pricing

Firms set the price for domestic sales in the domestic currency, but prices for sales abroad can be set in different currencies. Specifically, a firm located in country j sets a price $\tilde{P}_j^j(z)$ in its own currency for domestic sales. Its exports are invoiced in a basket of the three available currencies, with the weight of each being in the $[0,1]$ interval. The weights are denoted by γ with a subscript indicating the country of destination, as well as superscripts indicating the country of production and the currency of invoicing. Specifically $\gamma_i^{j,\text{ cur }k}$ is the share of currency k in the invoicing of its exports from country j to country i. The invoicing weights are exogenous and are the same for all firms in the exporting country.

The pricing by the firm producing brand z in country j and exporting to country i is represented by fixing a price $\tilde{P}_i^j(z)$ such that the price paid by the consumer in her own currency, i, is:

$$P_i^j(z) = \tilde{P}_i^j(z) \sum_{k=A,B,C} \left(\frac{S_k}{S_i}\right)^{\gamma_i^{j,\text{ cur }k}} = \tilde{P}_i^j(z)\,(S_i)^{-1}\,(S_B)^{\gamma_i^{j,\text{ cur }B}}\,(S_C)^{\gamma_i^{j,\text{ cur }C}} \tag{6}$$

where S_j is the exchange rate between currency A and currency j. It is expressed as the amount of currency A per unit of currency j, so an increase corresponds to a depreciation of currency A. The exchange rate between currency i and currency k, in terms of the amount of currency i per unit of currency k, is then given by S_k/S_i. The case of producer currency pricing (PCP) corresponds to $\gamma_i^{j,\text{ cur }j} = 1$ and all other weights being zero, while the case of local currency pricing (LCP) corresponds to $\gamma_i^{j,\text{ cur }i} = 1$ and all other weights being zero. Pricing in a vehicle currency (VCP) corresponds to $\gamma_i^{j,\text{ cur }j} = \gamma_i^{j,\text{ cur }i} = 0$ and $\gamma_i^{j,\text{ cur }k(\neq i,j)} = 1$.

Using (6) the local currency prices of goods produced in country A are:

$$\begin{aligned}
P_B^A(z) &= \tilde{P}_B^A(z)\,(S_B)^{\gamma_B^{i,\text{ cur }B}-1}\,(S_C)^{\gamma_B^{A,\text{ cur }C}} \\
P_C^A(z) &= \tilde{P}_C^A(z)\,(S_B)^{\gamma_C^{A,\text{ cur }B}}\,(S_C)^{\gamma_C^{A,\text{ cur }C}-1}
\end{aligned}$$

Similarly, the local currency prices of goods produced in country B are:

$$\begin{aligned}
P_A^B(z) &= \tilde{P}_A^B(z)\,(S_B)^{\gamma_A^{B,\text{ cur }B}}\,(S_C)^{\gamma_A^{B,\text{ cur }C}} \\
P_C^B(z) &= \tilde{P}_C^B(z)\,(S_B)^{\gamma_C^{B,\text{ cur }B}}\,(S_C)^{\gamma_C^{B,\text{ cur }C}-1}
\end{aligned}$$

And the local currency prices of goods produced in country C are:

$$P_A^C(z) = \tilde{P}_A^C(z)(S_B)^{\gamma_A^{C\text{ cur }B}}(S_C)^{\gamma_A^{C\text{ cur }C}}$$
$$P_B^C(z) = \tilde{P}_B^C(z)(S_B)^{\gamma_B^{C,\text{ cur }B}-1}(S_C)^{\gamma_B^{C\text{ cur }C}}$$

1.4 Five cases of invoicing

We illustrate our results by considering five cases for the pricing structure. The first case, referred to as PCP-SYM, is the situation where producer currency pricing applies to all trade flows, so there is always full exchange rate pass-through. The pricing parameters are then (for brevity we only report the invoicing parameters that are not zero):

$$\gamma_B^{A,\text{ cur}A} = \gamma_C^{A,\text{ cur}A} = 1$$
$$\gamma_A^{B,\text{ cur}B} = \gamma_C^{B,\text{ cur}B} = 1$$
$$\gamma_A^{C,\text{ cur}C} = \gamma_B^{C,\text{ cur}C} = 1$$

The second case, referred to as LCP-SYM, is the situation where local currency pricing applies to all trade flows, so there is never any exchange rate pass-through. The pricing parameters are then:

$$\gamma_B^{A,\text{ cur}B} = \gamma_C^{A,\text{ cur}C} = 1$$
$$\gamma_A^{B,\text{ cur}A} = \gamma_C^{B,\text{ cur}C} = 1$$
$$\gamma_A^{C,\text{ cur}A} = \gamma_B^{C,\text{ cur}B} = 1$$

In the last three cases the currency A (that we refer to as the dollar) is used in all trade flows that involve country A. There are three variants depending on the invoicing of trade between the periphery countries. In the DOL-PCP case trade flows between country B and C are invoiced in producer currency, and the pricing parameters are:

$$\gamma_B^{A,\text{ cur}A} = \gamma_C^{A,\text{ cur}A} = 1$$
$$\gamma_A^{B,\text{ cur}A} = \gamma_C^{B,\text{ cur}B} = 1$$
$$\gamma_A^{C,\text{ cur}A} = \gamma_B^{C,\text{ cur}C} = 1$$

In the DOL-LCP case trade flows between country B and C are invoiced in local currency, and the pricing parameters are:

$$\gamma_B^{A,\text{ cur}A} = \gamma_C^{A,\text{ cur}A} = 1$$
$$\gamma_A^{B,\text{ cur}A} = \gamma_C^{B,\text{ cur}C} = 1$$
$$\gamma_A^{C,\text{ cur}A} = \gamma_B^{C,\text{ cur}B} = 1$$

In the DOL-DOL case trade flows between country B and C are invoiced in dollar, and the pricing parameters are:

$$\begin{aligned} \gamma_B^{A,\,\mathrm{cur}A} &= \gamma_C^{A,\,\mathrm{cur}A} = 1 \\ \gamma_A^{B,\,\mathrm{cur}A} &= \gamma_C^{B,\,\mathrm{cur}A} = 1 \\ \gamma_A^{C,\,\mathrm{cur}A} &= \gamma_B^{C,\,\mathrm{cur}A} = 1 \end{aligned}$$

1.5 Firms' technology and output

Firms use a simple technology with constant returns to scale, subject to country-wide productivity shocks:

$$Y_i(z) = K_i H_i(z) \qquad i = A, B, C \tag{7}$$

The outputs are given by aggregating the various demands. Using the pricing structure detailed above, the output of a representative firm in country A is:

$$\begin{aligned} Y_A(z) &= \frac{1}{2} C_A^A(z) + \frac{1}{4} C_B^A(z) + \frac{1}{4} C_C^A(z) \\ &= \alpha \left[\frac{\tilde{P}_A^A(z)}{P_A^A} \right]^{-\lambda} \frac{P_A}{P_A^A} C_A \\ &\quad + \frac{1-\alpha}{2} \left[\frac{\tilde{P}_B^A(z) (S_B)^{\gamma_B^{i,\,\mathrm{cur}\,B}-1} (S_C)^{\gamma_B^{A,\,\mathrm{cur}\,C}}}{P_B^A} \right]^{-\lambda} \frac{P_B}{P_B^A} C_B \\ &\quad + \frac{1-\alpha}{2} \left[\frac{\tilde{P}_C^A(z) (S_B)^{\gamma_C^{A,\,\mathrm{cur}\,B}} (S_C)^{\gamma_C^{i,\,\mathrm{cur}\,C}-1}}{P_C^A} \right]^{-\lambda} \frac{P_C}{P_C^A} C_C \end{aligned} \tag{8}$$

The output of a representative firm in country B is:

$$
\begin{aligned}
Y_B(z) &= \frac{1}{2}C_A^B(z) + \frac{1}{4}C_B^B(z) + \frac{1}{4}C_C^B(z) \\
&= (1-\alpha)\left[\frac{\tilde{P}_A^B(z)\,(S_B)^{\gamma_A^{B,\text{ cur }B}}\,(S_C)^{\gamma_A^{B,\text{ cur }C}}}{P_A^B}\right]^{-\lambda}\frac{P_A}{P_A^B}C_A \qquad (9)\\
&+\frac{\alpha}{2}\left[\frac{\tilde{P}_B^B(z)}{P_B^B}\right]^{-\lambda}\frac{P_B}{P_B^B}C_B \\
&+\frac{\alpha}{2}\left[\frac{\tilde{P}_C^B(z)\,(S_B)^{\gamma_C^{B,\text{ cur }B}}\,(S_C)^{\gamma_C^{B,\text{ cur }C}-1}}{P_C^B}\right]^{-\lambda}\frac{P_C}{P_C^B}C_C
\end{aligned}
$$

The output of a representative firm in country C is:

$$
\begin{aligned}
Y_C(z) &= \frac{1}{2}C_A^C(z) + \frac{1}{4}C_B^C(z) + \frac{1}{4}C_C^C(z) \\
&= (1-\alpha)\left[\frac{\tilde{P}_A^C(z)\,(S_B)^{\gamma_A^{C\text{ cur }B}}\,(S_C)^{\gamma_A^{C\text{ cur }C}}}{P_A^C}\right]^{-\lambda}\frac{P_A}{P_A^C}C_A \qquad (10)\\
&+\frac{\alpha}{2}\left[\frac{\tilde{P}_B^C(z)\,(S_B)^{\gamma_B^{C,\text{ cur }B}-1}\,(S_C)^{\gamma_B^{C\text{ cur }C}}}{P_B^C}\right]^{-\lambda}\frac{P_B}{P_B^C}C_B \\
&+\frac{\alpha}{2}\left[\frac{\tilde{P}_C^C(z)}{P_C^C}\right]^{-\lambda}\frac{P_C}{P_C^C}C_C
\end{aligned}
$$

In equilibrium all firms in a given country are identical, so (8)-(10) are written in terms of per capita output as:

$$
\begin{aligned}
Y_A &= \alpha\frac{P_A}{P_A^A}C_A + \frac{1-\alpha}{2}\left[\frac{P_B}{P_B^A}C_B + \frac{P_C}{P_C^A}C_C\right] \qquad (11)\\
Y_B &= (1-\alpha)\frac{P_A}{P_A^B}C_A + \frac{\alpha}{2}\left[\frac{P_B}{P_B^B}C_B + \frac{P_C}{P_C^B}C_C\right] \qquad (12)\\
Y_C &= (1-\alpha)\frac{P_A}{P_A^C}C_A + \frac{\alpha}{2}\left[\frac{P_B}{P_B^C}C_B + \frac{P_C}{P_C^C}C_C\right] \qquad (13)
\end{aligned}
$$

1.6 Solution for the exchange rates

We abstract from government spending and assumes that the seigniorage income from monetary creation is repaid to the domestic households as a lump sum income ($M_i = -T_i$). The budget constraint (4) then implies that in each country the revenue of firms (the sum of profits and wages, denoted by REV) is equal to nominal consumption, which is itself linked to the money supply through the money demand (5):

$$REV_i = P_i C_i = \frac{1}{\chi} M_i$$

The revenues, expressed in the producer's currencies, are the output (11)-(13) multiplied by the relevant prices and exchange rates:

$$\begin{aligned}
REV_A &= \alpha P_A C_A + \frac{1-\alpha}{2} S_B P_B C_B + \frac{1-\alpha}{2} S_C P_C C_C \\
REV_B &= (1-\alpha) \frac{P_A C_A}{S_B} + \frac{\alpha}{2} P_B C_B + \frac{\alpha}{2} \frac{S_C}{S_B} P_C C_C \\
REV_C &= (1-\alpha) \frac{P_A C_A}{S_C} + \frac{\alpha}{2} \frac{S_B P_B C_B}{S_C} + \frac{\alpha}{2} P_C C_C
\end{aligned}$$

Combining these two sets of equations to substitute for the revenues, we get that the exchange rates are simply the ratios of the money supplies adjusted for the money demand shocks:

$$S_B = \frac{M_A}{M_B} \quad ; \quad S_C = \frac{M_A}{M_C} \tag{14}$$

Under complete disconnect between the center and the periphery ($\alpha = 1$), these equations only give the intra-periphery exchange rate S_B/S_C, and the center-periphery exchange rates S_B and S_C are not defined.[2] Throughout the analysis we consider the limit case where α approaches 1, and refer to it as the disconnect case for brevity.

2 Solution under flexible prices

2.1 Optimal prices

A useful benchmark is given by the case where firms can adjust their prices following the realization of shocks and the reaction by monetary au-

[2] In our setup exchange rates are fully determined by the relative monetary stances, a feature that is common to the various contributions in the literature. As a result, the model generates an exchange rate volatility that is well below the one observed in the data . This shortcoming can be adressed by introducing shocks to the money demands, with no impact on the message of our paper.

thorities. A representative firm in country A sets three prices, $P_A^A(z)$, $P_B^A(z)$ and $P_C^A(z)$ to maximize its profits:

$$\begin{aligned}
\Pi_A(z) &= \left(P_A^A(z) - \frac{W_A}{K_A}\right)\alpha\left[\frac{P_A^A(z)}{P_A^A}\right]^{-\lambda}\frac{P_A}{P_A^A}C_A \\
&\quad + \left(S_B P_B^A(z) - \frac{W_A}{K_A}\right)\frac{1-\alpha}{2}\left[\frac{P_B^A(z)}{P_B^A}\right]^{-\lambda}\frac{P_B}{P_B^A}C_B \\
&\quad + \left(S_C P_C^A(z) - \frac{W_A}{K_A}\right)\frac{1-\alpha}{2}\left[\frac{P_C^A(z)}{P_C^A}\right]^{-\lambda}\frac{P_C}{P_C^A}C_C
\end{aligned}$$

This is maximized by the following prices:

$$P_A^A = S_B P_B^A = S_C P_C^A = \frac{\lambda}{\lambda-1}(W_A/K_A) \tag{15}$$

where we used the fact that all firms in a given country set identical prices in equilibrium, so for instance $P_C^A(z) = P_C^A$. (15) shows that the law of one price holds, and all prices are set as a markup over marginal cost.

The optimal pricing by firms in country B and country C leads to similar expressions:

$$\frac{P_A^B}{S_B} = P_B^B = \frac{S_C P_C^B}{S_B} = \frac{\lambda}{\lambda-1}(W_B/K_B) \tag{16}$$

$$\frac{P_A^C}{S_C} = \frac{S_B P_B^C}{S_C} = P_C^C = \frac{\lambda}{\lambda-1}(W_C/K_C) \tag{17}$$

2.2 Output, consumption and welfare

The consumer price indexes are given by (1)-(3). Using the exchange rates (14), the labor supplies and the money demands (5) we get:

$$\begin{aligned}
P_A &= \frac{\lambda\kappa}{\lambda-1}\frac{1}{\chi}M_A(K_A)^{-\alpha}(K_B K_C)^{-\frac{1-\alpha}{2}} \\
P_B &= \frac{\lambda\kappa}{\lambda-1}\frac{1}{\chi}M_B(K_A)^{-(1-\alpha)}(K_B K_C)^{-\frac{\alpha}{2}} \\
P_C &= \frac{\lambda\kappa}{\lambda-1}\frac{1}{\chi}M_C(K_A)^{-(1-\alpha)}(K_B K_C)^{-\frac{\alpha}{2}}
\end{aligned}$$

Consumptions are computed from the money demands (5):

$$C_A = \frac{M_A}{\chi P_A} = \frac{\lambda - 1}{\lambda\kappa}(K_A)^{\alpha}(K_B K_C)^{\frac{1-\alpha}{2}} \tag{18}$$

$$C_B = C_C = \frac{\lambda - 1}{\lambda\kappa}(K_A)^{1-\alpha}(K_B K_C)^{\frac{\alpha}{2}} \tag{19}$$

The outputs are computed from (11)-(13) as:

$$Y_i = \frac{\lambda - 1}{\lambda\kappa} K_i \tag{20}$$

(20) show that productivity shocks are transmitted to output one-for-one and the amount of hours worked is unaffected.

We assume that productivity shocks are log-normal, with mean zero. Using (18)-(19) and (20), the welfare, abstracting from the direct impact of real balances, are then:

$$\begin{aligned} U_A &= E[\ln(C_A) - \kappa H_A] = E\ln(C_A) - \frac{\lambda-1}{\lambda} \\ &= \alpha E\ln(K_A) + \frac{1-\alpha}{2}E[\ln(K_B) + \ln(K_C)] + \Phi \\ &= \Phi \qquad (21) \\ U_B &= U_C = \Phi \qquad (22) \end{aligned}$$

where $\Phi = \ln\left(\frac{\lambda-1}{\lambda\kappa}\right) - \frac{\lambda-1}{\lambda}$. (21)-(22) shows that all countries face the same welfare under flexible prices.

3 Solution under preset prices

3.1 Optimal pricing and expected effort

A firm in country A sets its prices to maximize its expected discounted profits, with the marginal utility of consumption of domestic agents representing the discount factor:

$$
\begin{aligned}
E\frac{\Pi_A(z)}{P_A C_A} &= E\frac{1}{P_A C_A}\left(\tilde{P}_A^A(z) - \frac{W_A}{K_A}\right)\alpha\left[\frac{\tilde{P}_A^A(z)}{P_A^A}\right]^{-\lambda}\frac{P_A}{P_A^A}C_A \\
&+E\frac{1}{P_A C_A}\left(\tilde{P}_B^A(z)(S_B)^{\gamma_B^{i,\text{ cur }B}}(S_C)^{\gamma_B^{A,\text{ cur }C}} - \frac{W_A}{K_A}\right) \\
&\times\frac{1-\alpha}{2}\left[\frac{\tilde{P}_B^A(z)(S_B)^{\gamma_B^{i,\text{ cur }B}-1}(S_C)^{\gamma_B^{A,\text{ cur }C}}}{P_B^A}\right]^{-\lambda}\frac{P_B}{P_B^A}C_B \\
&+E\frac{1}{P_A C_A}\left(\tilde{P}_C^A(z)(S_B)^{\gamma_C^{A,\text{ cur }B}}(S_C)^{\gamma_C^{i,\text{ cur }C}} - \frac{W_A}{K_A}\right) \\
&\times\frac{1-\alpha}{2}\left[\frac{\tilde{P}_C^A(z)(S_B)^{\gamma_C^{A,\text{ cur }B}}(S_C)^{\gamma_C^{i,\text{ cur }C}-1}}{P_C^A}\right]^{-\lambda}\frac{P_C}{P_C^A}C_C
\end{aligned}
$$

This is maximized by the following prices:

$$
\begin{aligned}
\tilde{P}_A^A &= \frac{\lambda\kappa}{\lambda-1}\frac{1}{\chi}E\frac{M_A}{K_A} \\
\tilde{P}_B^A &= \frac{\lambda\kappa}{\lambda-1}\frac{1}{\chi}E\frac{1}{K_A}(M_A)^{\gamma_B^{A,\text{ cur }A}}(M_B)^{\gamma_B^{i,\text{ cur }B}}(M_C)^{\gamma_B^{A,\text{ cur }C}} \\
\tilde{P}_C^A &= \frac{\lambda\kappa}{\lambda-1}\frac{1}{\chi}E\frac{1}{K_A}(M_A)^{\gamma_C^{A,\text{ cur }A}}(M_B)^{\gamma_C^{A,\text{ cur }B}}(M_C)^{\gamma_C^{i,\text{ cur }C}}
\end{aligned}
\tag{23}
$$

where we used the exchange rate solution (14), the labor supplies and the money demands (5), and the fact that all firms in a given country set identical prices in equilibrium.

Following similar steps, the optimal prices for a firm in country B are:

$$
\begin{aligned}
\tilde{P}_A^B &= \frac{\lambda\kappa}{\lambda-1}\frac{1}{\chi}E\frac{1}{K_B}(M_A)^{\gamma_A^{B,\text{ cur }A}}(M_B)^{\gamma_A^{B,\text{ cur }B}}(M_C)^{\gamma_A^{B,\text{ cur }C}} \\
\tilde{P}_B^B &= \frac{\lambda\kappa}{\lambda-1}\frac{1}{\chi}E\frac{M_B}{K_B} \\
\tilde{P}_C^B &= \frac{\lambda\kappa}{\lambda-1}\frac{1}{\chi}E\frac{1}{K_B}(M_A)^{\gamma_C^{B,\text{ cur }A}}(M_B)^{\gamma_C^{B,\text{ cur }B}}(M_C)^{\gamma_C^{B,\text{ cur }C}}
\end{aligned}
\tag{24}
$$

Similarly, the optimal prices for a firm in country C are:

$$\begin{aligned}
\tilde{P}_A^C &= \frac{\lambda\kappa}{\lambda-1}\frac{1}{\chi}E\frac{1}{K_C}\left(M_A\right)^{\gamma_A^{C,\text{ cur }A}}\left(M_B\right)^{\gamma_A^{C\text{ cur }B}}\left(M_C\right)^{\gamma_A^{C\text{ cur }C}} \\
\tilde{P}_B^C &= \frac{\lambda\kappa}{\lambda-1}\frac{1}{\chi}E\frac{1}{K_C}\left(M_A\right)^{\gamma_B^{C,\text{ cur }A}}\left(M_B\right)^{\gamma_B^{C,\text{ cur }B}}\left(M_C\right)^{\gamma_B^{C\text{ cur }C}} \qquad (25) \\
\tilde{P}_C^C &= \frac{\lambda\kappa}{\lambda-1}\frac{1}{\chi}E\frac{M_C}{K_C}
\end{aligned}$$

(23)-(25) show that the preset prices are markups over the expectation of ratios of weighted monetary stances and the productivity faced by the firm. (23)-(25) can be written in a more general form as:

$$\begin{aligned}
\tilde{P}_i^j &= \frac{\lambda\kappa}{\lambda-1}\frac{1}{\chi}E\frac{1}{K_j}\left(M_A\right)^{\gamma_i^{j,\text{ cur }A}}\left(M_B\right)^{\gamma_i^{j,\text{ cur }B}}\left(M_C\right)^{\gamma_i^{j\text{ cur }C}} \qquad (26) \\
\gamma_j^{j,\text{ cur }j} &= 1 \qquad \gamma_j^{j,\text{ cur }k\neq j} = 0
\end{aligned}$$

Substituting the optimal prices (23)-(25) in the output demands (11)-(13) we can show that the expected effort is not affected by monetary policy in any country:

$$E\frac{Y_i}{K_i} = \frac{\lambda-1}{\lambda\kappa} \qquad i = A, B, C \qquad (27)$$

3.2 Consumption

Consumption is driven by the money demands (5). Using the consumer price indexes (1)-(3), the exchange rate (14) and the pass-through structure (6), we write consumption in country A as:

$$\begin{aligned}
C_A &= \frac{M_A}{\chi P_A} = \frac{M_A}{\chi\left(\tilde{P}_A^A\right)^{\alpha}\left(P_A^B P_A^C\right)^{\frac{1-\alpha}{2}}} \\
&= \frac{1}{\chi}\left(\tilde{P}_A^A\right)^{-\alpha}\left(\tilde{P}_A^B\tilde{P}_A^C\right)^{-\frac{1-\alpha}{2}}\left(M_A\right)^{\alpha+\frac{1-\alpha}{2}\left(\gamma_A^{B,\text{ cur }A}+\gamma_A^{C,\text{ cur }A}\right)} \qquad (28) \\
&\quad \left(M_B\right)^{\frac{1-\alpha}{2}\left(\gamma_A^{B,\text{ cur }B}+\gamma_A^{C\text{ cur }B}\right)}\left(M_C\right)^{\frac{1-\alpha}{2}\left(\gamma_A^{B,\text{ cur }C}+\gamma_A^{C\text{ cur }C}\right)}
\end{aligned}$$

Consumption in country B is:

$$\begin{aligned}
C_B &= \frac{1}{\chi}\left(\tilde{P}_B^A\right)^{-(1-\alpha)}\left(\tilde{P}_B^B\tilde{P}_B^C\right)^{-\frac{\alpha}{2}}\left(M_A\right)^{(1-\alpha)\gamma_B^{A,\text{ cur }A}+\frac{\alpha}{2}\gamma_B^{C,\text{ cur }A}} \\
&\quad \left(M_B\right)^{\frac{\alpha}{2}+(1-\alpha)\gamma_B^{A,\text{ cur }B}+\frac{\alpha}{2}\gamma_B^{C,\text{ cur }B}}\left(M_C\right)^{(1-\alpha)\gamma_B^{A,\text{ cur }C}+\frac{\alpha}{2}\gamma_B^{C\text{ cur }C}}
\end{aligned}$$

and consumption in country C is:

$$C_C = \frac{1}{\chi}\left(\tilde{P}_C^A\right)^{-(1-\alpha)}\left(\tilde{P}_C^B\tilde{P}_C^C\right)^{-\frac{\alpha}{2}}(M_A)^{(1-\alpha)\gamma_C^{A,\text{ cur }A}+\frac{\alpha}{2}\gamma_C^{B,\text{ cur }A}}$$
$$(M_B)^{(1-\alpha)\gamma_C^{A,\text{ cur }B}+\frac{\alpha}{2}\gamma_C^{B,\text{ cur }B}}(M_C)^{\frac{\alpha}{2}+(1-\alpha)\gamma_C^{i,\text{ cur }C}+\frac{\alpha}{2}\gamma_C^{B,\text{ cur }C}}$$

The next step is to substitute for the preset prices using (23)-(25). Consumption in country A (28) then becomes:

$$\begin{aligned} C_A = {} & \frac{\lambda-1}{\lambda\kappa}(M_A)^{\alpha+\frac{1-\alpha}{2}\left(\gamma_A^{C,\text{ cur }A}+\gamma_A^{B,\text{ cur }A}\right)}(M_B)^{\frac{1-\alpha}{2}\left(\gamma_A^{B,\text{ cur }B}+\gamma_A^{C\text{ cur }B}\right)} \\ & (M_C)^{\frac{1-\alpha}{2}\left(\gamma_A^{B,\text{ cur }C}+\gamma_A^{C\text{ cur }C}\right)}\left[E\frac{M_A}{K_A}\right]^{-\alpha} \\ & \left[E\frac{1}{K_B}(M_A)^{\gamma_A^{B,\text{ cur }A}}(M_B)^{\gamma_A^{B,\text{ cur }B}}(M_C)^{\gamma_A^{B,\text{ cur }C}}\right]^{-\frac{1-\alpha}{2}} \\ & \left[E\frac{1}{K_C}(M_A)^{\gamma_A^{C,\text{ cur }A}}(M_B)^{\gamma_A^{C\text{ cur }B}}(M_C)^{\gamma_A^{C\text{ cur }C}}\right]^{-\frac{1-\alpha}{2}} \end{aligned} \quad (29)$$

Consumption in country B is:

$$\begin{aligned} C_B = {} & \frac{\lambda-1}{\lambda\kappa}(M_A)^{(1-\alpha)\gamma_B^{A,\text{ cur }A}+\frac{\alpha}{2}\gamma_B^{C,\text{ cur }A}}(M_B)^{\frac{\alpha}{2}+(1-\alpha)\gamma_B^{A,\text{ cur }B}+\frac{\alpha}{2}\gamma_B^{C,\text{ cur }B}} \\ & (M_C)^{(1-\alpha)\gamma_B^{A,\text{ cur }C}+\frac{\alpha}{2}\gamma_B^{C\text{ cur }C}}\left[E\frac{M_B}{K_B}\right]^{-\frac{\alpha}{2}} \\ & \left[E\frac{1}{K_A}(M_A)^{\gamma_B^{A,\text{ cur }A}}(M_B)^{\gamma_B^{A,\text{ cur }B}}(M_C)^{\gamma_B^{A,\text{ cur }C}}\right]^{-(1-\alpha)} \\ & \left[E\frac{1}{K_C}(M_A)^{\gamma_B^{C,\text{ cur }A}}(M_B)^{\gamma_B^{C,\text{ cur }B}}(M_C)^{\gamma_B^{C\text{ cur }C}}\right]^{-\frac{\alpha}{2}} \end{aligned} \quad (30)$$

Consumption in country C is:

$$\begin{aligned} C_C = {} & \frac{\lambda-1}{\lambda\kappa}(M_A)^{(1-\alpha)\gamma_C^{A,\text{ cur }A}+\frac{\alpha}{2}\gamma_C^{B,\text{ cur }A}}(M_B)^{(1-\alpha)\gamma_C^{A,\text{ cur }B}+\frac{\alpha}{2}\gamma_C^{B,\text{ cur }B}} \\ & (M_C)^{\frac{\alpha}{2}+(1-\alpha)\gamma_C^{A,\text{ cur }C}+\frac{\alpha}{2}\gamma_C^{B,\text{ cur }C}}\left[E\frac{M_C}{K_C}\right]^{-\frac{\alpha}{2}} \\ & \left[E\frac{1}{K_A}(M_A)^{\gamma_C^{A,\text{ cur }A}}(M_B)^{\gamma_C^{A,\text{ cur }B}}(M_C)^{\gamma_C^{A,\text{ cur }C}}\right]^{-(1-\alpha)} \\ & \left[E\frac{1}{K_B}(M_A)^{\gamma_C^{B,\text{ cur }A}}(M_B)^{\gamma_C^{B,\text{ cur }B}}(M_C)^{\gamma_C^{B,\text{ cur }C}}\right]^{-\frac{\alpha}{2}} \end{aligned} \quad (31)$$

3.3 Impact of monetary stances on utility

From (27) expected effort is unaffected by shocks and monetary policy in all countries. The welfare then boils down to the expected log of the consumptions (29)-(31). The impact of monetary policy is computed by first taking derivatives of the welfare with respect to the monetary stance in a state s. In terms of the welfare in country A, the impact of the monetary stance in country A is:

$$\begin{aligned}
&\frac{\partial E\ln(C_A)}{\partial M_{A,s}} \\
= \; & \left[\alpha + \frac{1-\alpha}{2}\left(\gamma_A^{C,\text{ cur }A} + \gamma_A^{B,\text{ cur }A}\right)\right]\pi_s\frac{1}{M_{A,s}} - \pi_s\frac{1}{M_{A,s}}\alpha\frac{\frac{M_{A,s}}{K_{A,s}}}{E\frac{M_A}{K_A}} \\
& -\pi_s\frac{1}{M_{A,s}}\frac{1-\alpha}{2}\gamma_A^{B,\text{ cur }A}\frac{\frac{1}{K_{B,s}}(M_{A,s})^{\gamma_A^{B,\text{ cur }A}}(M_{B,s})^{\gamma_A^{B,\text{ cur }B}}(M_{C,s})^{\gamma_A^{B,\text{ cur }C}}}{E\frac{1}{K_B}(M_A)^{\gamma_A^{B,\text{ cur }A}}(M_B)^{\gamma_A^{B,\text{ cur }B}}(M_C)^{\gamma_A^{B,\text{ cur }C}}} \\
& -\pi_s\frac{1}{M_{A,s}}\frac{1-\alpha}{2}\gamma_A^{C,\text{ cur }A}\frac{\frac{1}{K_{C,s}}(M_{A,s})^{\gamma_A^{C,\text{ cur }A}}(M_{B,s})^{\gamma_A^{C\text{ cur }B}}(M_{C,s})^{\gamma_A^{C\text{ cur }C}}}{E\frac{1}{K_C}(M_A)^{\gamma_A^{C,\text{ cur }A}}(M_B)^{\gamma_A^{C,\text{ cur }B}}(M_C)^{\gamma_A^{C\text{ cur }C}}}
\end{aligned}$$

where s is an index of the state of nature, and π_s is the probability of that state. This can be simplified by writing the expression in terms of log deviations from the deterministic steady state, denoted by San-Serif variables. Recall that the expected log productivity is zero: $E\mathsf{k}_i = 0$. As shown below, the log of monetary stances are a linear functions of the log productivities, hence their expected value is also zero: $E\mathsf{m}_i = 0$. The derivative then becomes:

$$\begin{aligned}
&\frac{\partial E\ln(C_A)}{\pi_s\partial M_{A,s}} \\
= \; & -\alpha\left(\mathsf{m}_{A,s} - \mathsf{k}_{A,s}\right) \\
& -\frac{1-\alpha}{2}\gamma_A^{B,\text{ cur }A}\left[\gamma_A^{B,\text{ cur }A}\mathsf{m}_{A,s} + \gamma_A^{B,\text{ cur }B}\mathsf{m}_{B,s} + \gamma_A^{B,\text{ cur }C}\mathsf{m}_{C,s} - \mathsf{k}_{B,s}\right] \\
& -\frac{1-\alpha}{2}\gamma_A^{C,\text{ cur }A}\left[\gamma_A^{C,\text{ cur }A}\mathsf{m}_{A,s} + \gamma_A^{C\text{ cur }B}\mathsf{m}_{B,s} + \gamma_A^{C\text{ cur }C}\mathsf{m}_{C,s} - \mathsf{k}_{C,s}\right]
\end{aligned}$$

Following similar steps the impact of the monetary stance in country B is:

$$\begin{aligned}
&\frac{\partial E \ln\left(C_A\right)}{\pi_s \partial M_{B,s}} \\
= \;& -\frac{1-\alpha}{2}\gamma_A^{B,\text{ cur } B}\left[\gamma_A^{B,\text{ cur } A}\mathsf{m}_{A,s}+\gamma_A^{B,\text{ cur } B}\mathsf{m}_{B,s}+\gamma_A^{B,\text{ cur } C}\mathsf{m}_{C,s}-\mathsf{k}_{B,s}\right] \\
& -\frac{1-\alpha}{2}\gamma_A^{C,\text{ cur } B}\left[\gamma_A^{C,\text{ cur } A}\mathsf{m}_{A,s}+\gamma_A^{C,\text{ cur } B}\mathsf{m}_{B,s}+\gamma_A^{C\text{ cur } C}\mathsf{m}_{C,s}-\mathsf{k}_{C,s}\right]
\end{aligned}$$

And the impact of the monetary stance in country C is:

$$\begin{aligned}
&\frac{\partial E \ln\left(C_A\right)}{\pi_s \partial M_{C,s}} \\
= \;& -\frac{1-\alpha}{2}\gamma_A^{B,\text{ cur } C}\left[\gamma_A^{B,\text{ cur } A}\mathsf{m}_{A,s}+\gamma_A^{B,\text{ cur } B}\mathsf{m}_{B,s}+\gamma_A^{B,\text{ cur } C}\mathsf{m}_{C,s}-\mathsf{k}_{B,s}\right] \\
& -\frac{1-\alpha}{2}\gamma_A^{C\text{ cur } C}\left[\gamma_A^{C,\text{ cur } A}\mathsf{m}_{A,s}+\gamma_A^{C,\text{ cur } B}\mathsf{m}_{B,s}+\gamma_A^{C\text{ cur } C}\mathsf{m}_{C,s}-\mathsf{k}_{C,s}\right]
\end{aligned}$$

Turning to the welfare in country B, the impact of the monetary stance in country A is:

$$\begin{aligned}
&\frac{\partial E \ln\left(C_B\right)}{\pi_s \partial M_{A,s}} \\
= \;& -(1-\alpha)\gamma_B^{A,\text{ cur } A}\left[\gamma_B^{A,\text{ cur } A}\mathsf{m}_{A,s}+\gamma_B^{A,\text{ cur } B}\mathsf{m}_{B,s}+\gamma_B^{A,\text{ cur } C}\mathsf{m}_{C,s}-\mathsf{k}_{A,s}\right] \\
& -\frac{\alpha}{2}\gamma_B^{C\text{ cur } A}\left[\gamma_B^{C\text{ cur } A}\mathsf{m}_{A,s}+\gamma_B^{C,\text{ cur } B}\mathsf{m}_{B,s}+\gamma_B^{C\text{ cur } C}\mathsf{m}_{C,s}-\mathsf{k}_{C,s}\right]
\end{aligned}$$

The impact of the monetary stance in country B is:

$$\begin{aligned}
&\frac{\partial E \ln\left(C_B\right)}{\pi_s \partial M_{B,s}} \\
= \;& -(1-\alpha)\gamma_B^{A,\text{ cur } B}\left[\gamma_B^{A,\text{ cur } A}\mathsf{m}_{A,s}+\gamma_B^{A,\text{ cur } B}\mathsf{m}_{B,s}+\gamma_B^{A,\text{ cur } C}\mathsf{m}_{C,s}-\mathsf{k}_{A,s}\right] \\
& -\frac{\alpha}{2}\left(\mathsf{m}_{B,s}-\mathsf{k}_{B,s}\right) \\
& -\frac{\alpha}{2}\gamma_B^{C,\text{ cur } B}\left[\gamma_B^{C\text{ cur } A}\mathsf{m}_{A,s}+\gamma_B^{C,\text{ cur } B}\mathsf{m}_{B,s}+\gamma_B^{C\text{ cur } C}\mathsf{m}_{C,s}-\mathsf{k}_{C,s}\right]
\end{aligned}$$

The impact of the monetary stance in country C is:

$$\begin{aligned}
&\frac{\partial E \ln (C_B)}{\pi_s \partial M_{C,s}} \\
= \; & -(1-\alpha)\, \gamma_B^{A,\text{ cur } C} \left[\gamma_B^{A,\text{ cur } A} \mathsf{m}_{A,s} + \gamma_B^{A,\text{ cur } B} \mathsf{m}_{B,s} + \gamma_B^{A,\text{ cur } C} \mathsf{m}_{C,s} - \mathsf{k}_{A,s} \right] \\
& -\frac{\alpha}{2} \gamma_B^{C \text{ cur } C} \left[\gamma_B^{C \text{ cur } A} \mathsf{m}_{A,s} + \gamma_B^{C,\text{ cur } B} \mathsf{m}_{B,s} + \gamma_B^{C \text{ cur } C} \mathsf{m}_{C,s} - \mathsf{k}_{C,s} \right]
\end{aligned}$$

Finally, consider to the welfare in country C. The impact of the monetary stance in country A is:

$$\begin{aligned}
&\frac{\partial E \ln (C_C)}{\pi_s \partial M_{A,s}} \\
= \; & -(1-\alpha)\, \gamma_C^{A,\text{ cur } A} \left[\gamma_C^{A,\text{ cur } A} \mathsf{m}_{A,s} + \gamma_C^{A,\text{ cur } B} \mathsf{m}_{B,s} + \gamma_C^{A,\text{ cur } C} \mathsf{m}_{C,s} - \mathsf{k}_{A,s} \right] \\
& -\frac{\alpha}{2} \gamma_C^{B,\text{ cur } A} \left[\gamma_C^{B,\text{ cur } A} \mathsf{m}_{A,s} + \gamma_C^{B,\text{ cur } B} \mathsf{m}_{B,s} + \gamma_C^{B,\text{ cur } C} \mathsf{m}_{C,s} - \mathsf{k}_{B,s} \right]
\end{aligned}$$

The impact of the monetary stance in country B is:

$$\begin{aligned}
&\frac{\partial E \ln (C_C)}{\pi_s \partial M_{B,s}} \\
= \; & -(1-\alpha)\, \gamma_C^{A,\text{ cur } B} \left[\gamma_C^{A,\text{ cur } A} \mathsf{m}_{A,s} + \gamma_C^{A,\text{ cur } B} \mathsf{m}_{B,s} + \gamma_C^{A,\text{ cur } C} \mathsf{m}_{C,s} - \mathsf{k}_{A,s} \right] \\
& -\frac{\alpha}{2} \gamma_C^{B,\text{ cur } B} \left[\gamma_C^{B,\text{ cur } A} \mathsf{m}_{A,s} + \gamma_C^{B,\text{ cur } B} \mathsf{m}_{B,s} + \gamma_C^{B,\text{ cur } C} \mathsf{m}_{C,s} - \mathsf{k}_{B,s} \right]
\end{aligned}$$

The impact of the monetary stance in country C is:

$$\begin{aligned}
&\frac{\partial E \ln (C_C)}{\pi_s \partial M_{C,s}} \\
= \; & -\frac{\alpha}{2} \left(\mathsf{m}_{C,s} - \mathsf{k}_{C,s} \right) \\
& -(1-\alpha)\, \gamma_C^{A,\text{ cur } C} \left[\gamma_C^{A,\text{ cur } A} \mathsf{m}_{A,s} + \gamma_C^{A,\text{ cur } B} \mathsf{m}_{B,s} + \gamma_C^{A,\text{ cur } C} \mathsf{m}_{C,s} - \mathsf{k}_{A,s} \right] \\
& -\frac{\alpha}{2} \gamma_C^{B,\text{ cur } C} \left[\gamma_C^{B,\text{ cur } A} \mathsf{m}_{A,s} + \gamma_C^{B,\text{ cur } B} \mathsf{m}_{B,s} + \gamma_C^{B,\text{ cur } C} \mathsf{m}_{C,s} - \mathsf{k}_{B,s} \right]
\end{aligned}$$

3.4 Welfare levels

The monetary authorities in each country react to productivity shocks, with the reaction function being linear in logs as shown below:

$$m_i = \delta_A^i k_A + \delta_B^i k_B + \delta_C^i k_C \qquad i = A,\, B,\, C \tag{32}$$

where logs are denoted by lower case letters. From (29) the expected log consumption consists of two main types of terms. First are expectation of the log of shocks:

$$\sum_{i=A,B,C} \vartheta_i E(k_i)$$

for some coefficients ϑ_i. As we take the shocks to be log-normal (at least up to a second order approximation) with mean zero, these terms are all equal to zero. The second terms are the logs of expectations, of the form:

$$\ln E\left[\prod_{i=A,B,C} (K_i)^{\hat{\vartheta}_i}\right] = \ln E\left[\prod_{i=A,B,C} \exp\left(\hat{\vartheta}_i k_i\right)\right] = \ln E\left[\exp\left(\sum_{i=A,B,C} \hat{\vartheta}_i k_i\right)\right]$$

for some coefficients $\hat{\vartheta}_i$. We recall the following property of the log normal-distribution:

$$E\left((X_i)^a\right) = E\left(\exp(ax_i)\right) = \exp\left(aEx_i + \frac{1}{2}a^2 Var(x_i)\right)$$

which implies:

$$\ln E\left[\prod_{i=A,B,C} (K_i)^{\hat{\vartheta}_i}\right] = \frac{1}{2} Var\left[\sum_{i=A,B,C} \hat{\vartheta}_i k_i\right]$$

We apply this property to the expected log of consumption in country A given by (29):

$$\begin{aligned}
E\ln C_A &= \ln\frac{\lambda-1}{\lambda\kappa} - \alpha\frac{1}{2}Var\left[\mathsf{m}_A - \mathsf{k}_A\right] \\
&\quad -\frac{1-\alpha}{2}\frac{1}{2}Var\left[\gamma_A^{B,\text{ cur }A}\mathsf{m}_A + \gamma_A^{B,\text{ cur }B}\mathsf{m}_B + \gamma_A^{B,\text{ cur }C}\mathsf{m}_C - \mathsf{k}_B\right] \\
&\quad -\frac{1-\alpha}{2}\frac{1}{2}Var\left[\gamma_A^{C,\text{ cur }A}\mathsf{m}_A + \gamma_A^{C\text{ cur }B}\mathsf{m}_B + \gamma_A^{C\text{ cur }C}\mathsf{m}_C - \mathsf{k}_C\right]
\end{aligned}$$

where we used the fact that the logs in the variances only have to be exact to a first order. We can then use:

$$\begin{aligned}
\mathsf{m}_A &= \delta_A^A\mathsf{k}_A + \delta_B^A\mathsf{k}_B + \delta_C^A\mathsf{k}_C \\
\mathsf{m}_B &= \delta_A^B\mathsf{k}_A + \delta_B^B\mathsf{k}_B + \delta_C^B\mathsf{k}_C \\
\mathsf{m}_C &= \delta_A^C\mathsf{k}_A + \delta_B^C\mathsf{k}_B + \delta_C^C\mathsf{k}_C
\end{aligned}$$

to write:

$$
\begin{aligned}
& E\ln C_A - \ln\frac{\lambda-1}{\lambda\kappa} \\
= \; & -\alpha\frac{1}{2}Var\left[\delta_A^A\mathsf{k}_A + \delta_B^A\mathsf{k}_B + \delta_C^A\mathsf{k}_C - \mathsf{k}_A\right] \\
& -\frac{1-\alpha}{2}\frac{1}{2}Var\left[\begin{array}{c} \gamma_A^{B,\text{ cur }A}\left(\delta_A^A\mathsf{k}_A + \delta_B^A\mathsf{k}_B + \delta_C^A\mathsf{k}_C\right) \\ +\gamma_A^{B,\text{ cur }B}\left(\delta_A^B\mathsf{k}_A + \delta_B^B\mathsf{k}_B + \delta_C^B\mathsf{k}_C\right) \\ +\gamma_A^{B,\text{ cur }C}\left(\delta_A^C\mathsf{k}_A + \delta_B^C\mathsf{k}_B + \delta_C^C\mathsf{k}_C\right) - \mathsf{k}_B \end{array}\right] \\
& -\frac{1-\alpha}{2}\frac{1}{2}Var\left[\begin{array}{c} \gamma_A^{C,\text{ cur }A}\left(\delta_A^A\mathsf{k}_A + \delta_B^A\mathsf{k}_B + \delta_C^A\mathsf{k}_C\right) \\ +\gamma_A^{C\text{ cur }B}\left(\delta_A^B\mathsf{k}_A + \delta_B^B\mathsf{k}_B + \delta_C^B\mathsf{k}_C\right) \\ +\gamma_A^{C\text{ cur }C}\left(\delta_A^C\mathsf{k}_A + \delta_B^C\mathsf{k}_B + \delta_C^C\mathsf{k}_C\right) - \mathsf{k}_C \end{array}\right]
\end{aligned} \tag{33}
$$

Following similar steps, we write the expected log consumption in country B as:

$$
\begin{aligned}
& E\ln C_B - \ln\frac{\lambda-1}{\lambda\kappa} \\
= \; & -(1-\alpha)\frac{1}{2}Var\left[\begin{array}{c} \gamma_B^{A,\text{ cur }A}\left(\delta_A^A\mathsf{k}_A + \delta_B^A\mathsf{k}_B + \delta_C^A\mathsf{k}_C\right) \\ +\gamma_B^{A,\text{ cur }B}\left(\delta_A^B\mathsf{k}_A + \delta_B^B\mathsf{k}_B + \delta_C^B\mathsf{k}_C\right) \\ +\gamma_B^{A,\text{ cur }C}\left(\delta_A^C\mathsf{k}_A + \delta_B^C\mathsf{k}_B + \delta_C^C\mathsf{k}_C\right) - \mathsf{k}_A \end{array}\right] \\
& -\frac{\alpha}{4}Var\left[\delta_A^B\mathsf{k}_A + \delta_B^B\mathsf{k}_B + \delta_C^B\mathsf{k}_C - \mathsf{k}_B\right] \\
& -\frac{\alpha}{4}Var\left[\begin{array}{c} \gamma_B^{C,\text{ cur }A}\left(\delta_A^A\mathsf{k}_A + \delta_B^A\mathsf{k}_B + \delta_C^A\mathsf{k}_C\right) \\ +\gamma_B^{C,\text{ cur }B}\left(\delta_A^B\mathsf{k}_A + \delta_B^B\mathsf{k}_B + \delta_C^B\mathsf{k}_C\right) \\ +\gamma_B^{C\text{ cur }C}\left(\delta_A^C\mathsf{k}_A + \delta_B^C\mathsf{k}_B + \delta_C^C\mathsf{k}_C\right) - \mathsf{k}_C \end{array}\right]
\end{aligned} \tag{34}
$$

The expected log consumption in country C is given by:

$$
\begin{aligned}
& E\ln C_C - \ln\frac{\lambda-1}{\lambda\kappa} \\
= \; & -(1-\alpha)\frac{1}{2}Var\left[\begin{array}{c} \gamma_C^{A,\text{ cur }A}\left(\delta_A^A\mathsf{k}_A + \delta_B^A\mathsf{k}_B + \delta_C^A\mathsf{k}_C\right) \\ +\gamma_C^{A,\text{ cur }B}\left(\delta_A^B\mathsf{k}_A + \delta_B^B\mathsf{k}_B + \delta_C^B\mathsf{k}_C\right) \\ +\gamma_C^{A,\text{ cur }C}\left(\delta_A^C\mathsf{k}_A + \delta_B^C\mathsf{k}_B + \delta_C^C\mathsf{k}_C\right) - \mathsf{k}_A \end{array}\right] \\
& -\frac{\alpha}{4}Var\left[\begin{array}{c} \gamma_C^{B,\text{ cur }A}\left(\delta_A^A\mathsf{k}_A + \delta_B^A\mathsf{k}_B + \delta_C^A\mathsf{k}_C\right) \\ +\gamma_C^{B,\text{ cur }B}\left(\delta_A^B\mathsf{k}_A + \delta_B^B\mathsf{k}_B + \delta_C^B\mathsf{k}_C\right) \\ +\gamma_C^{B,\text{ cur }C}\left(\delta_A^C\mathsf{k}_A + \delta_B^C\mathsf{k}_B + \delta_C^C\mathsf{k}_C\right) - \mathsf{k}_B \end{array}\right] \\
& -\frac{\alpha}{4}Var\left[\delta_A^C\mathsf{k}_A + \delta_B^C\mathsf{k}_B + \delta_C^C\mathsf{k}_C - \mathsf{k}_C\right]
\end{aligned} \tag{35}
$$

(33)-(35) show that the welfare is driven by productivity shocks, along with the response of policy to these shocks. The exact structure of invoicing affects the welfare impact of particular shocks.

In terms of welfare, the right-hand side of (33)-(35) also correspond to the difference between the welfare under sticky prices and the welfare under flexible prices (21)-(22), as expected effort is not affected by price rigidities.

4 Optimal monetary policy in a decentralized setting

4.1 General relations

We consider a decentralized allocation where each monetary authority maximizes the welfare of its own residents, taking the conduct of policy in other countries as given. The policies are then driven by the following conditions:

$$\frac{\partial E \ln (C_A)}{\pi_s \partial M_{A,s}} = \frac{\partial E \ln (C_B)}{\pi_s \partial M_{B,s}} = \frac{\partial E \ln (C_C)}{\pi_s \partial M_{C,s}} = 0$$

For simplicity, we focus on the fives particular cases of the pricing structure to illustrate the results.

4.2 Monetary rules

Under PCP-SYM the decentralized monetary policy is fully inward looking in each country:

$$\mathsf{m}_{i,s} = \mathsf{k}_{i,s} \qquad i = A,\ B,\ C \tag{36}$$

The exchange rate movements then reflects the bilateral productivity shocks.

Under LCP-SYM the monetary stances react to a weighted average of shocks, with the weights reflecting home bias:

$$\begin{aligned}
\mathsf{m}_{A,s} &= \alpha \mathsf{k}_{A,s} + (1-\alpha)\frac{\mathsf{k}_{B,s} + \mathsf{k}_{C,s}}{2} \\
\mathsf{m}_{B,s} &= (1-\alpha)\,\mathsf{k}_{A,s} + \alpha \frac{\mathsf{k}_{B,s} + \mathsf{k}_{C,s}}{2} \\
\mathsf{m}_{C,s} &= (1-\alpha)\,\mathsf{k}_{A,s} + \alpha \frac{\mathsf{k}_{B,s} + \mathsf{k}_{C,s}}{2}
\end{aligned} \tag{37}$$

Under any of the DOL- cases, monetary policy in country A reacts to a weighted average of shocks, exactly as under LCP-SYM:

$$\mathsf{m}_{A,s} = \alpha \mathsf{k}_{A,s} + (1-\alpha) \frac{\mathsf{k}_{B,s} + \mathsf{k}_{C,s}}{2} \tag{38}$$

Under DOL-PCP, monetary policy in the periphery is fully inward looking:

$$\mathsf{m}_{B,s} = \mathsf{k}_{B,s} \quad ; \quad \mathsf{m}_{C,s} = \mathsf{k}_{C,s} \tag{39}$$

Under DOL-LCP, monetary policy in the periphery react only to the average periphery shock:

$$\mathsf{m}_{B,s} = \frac{\mathsf{k}_{B,s} + \mathsf{k}_{C,s}}{2} \qquad \mathsf{m}_{C,s} = \frac{\mathsf{k}_{B,s} + \mathsf{k}_{C,s}}{2} \tag{40}$$

Under DOL-DOL, monetary policy in the periphery is fully inward looking:

$$\mathsf{m}_{B,s} = \mathsf{k}_{B,s} \quad ; \quad \mathsf{m}_{C,s} = \mathsf{k}_{C,s} \tag{41}$$

Not that if $\mathsf{k}_{B,s} = \mathsf{k}_{C,s}$ the DOL- setups are the same, and monetary policy in the periphery follows the shock in the periphery. This corresponds to a two-country center-periphery version of the model.

We can also compute the response of the worldwide average of monetary stances. Under the symmetric cases (PCP-SYM and LCP-SYM), it simply reflects the average of shocks:

$$\mathsf{m}_{W,s} = \frac{1}{2}\mathsf{m}_{A,s} + \frac{1}{2}\frac{\mathsf{m}_{B,s} + \mathsf{m}_{C,s}}{2} = \mathsf{k}_{W,s}$$

Under any DOL- cases it is smaller than the average of shocks when shocks are concentrated in country A. This is because country A reacts relatively little to its own shocks:

$$\mathsf{m}_{W,s} = \frac{\alpha}{2}\mathsf{k}_{A,s} + \frac{2-\alpha}{2}\frac{\mathsf{k}_{B,s} + \mathsf{k}_{C,s}}{2} = \mathsf{k}_{W,s} - \frac{1-\alpha}{2}\left(\mathsf{k}_{A,s} - \frac{\mathsf{k}_{B,s} + \mathsf{k}_{C,s}}{2}\right)$$

Note that all the monetary rules are unaffected by the variances of the various shocks.

4.3 Pegging to the center currency

Given the central role of the center currency in the invoicing decisions, we also consider a policy where each periphery country pegs its exchange rate to the center:

$$\mathsf{m}_{i,s} = \mathsf{m}_{A,s} \qquad i = B,\, C$$

The center's monetary policy is still set according to the following condition:

$$0 = \frac{\partial E \ln (C_A)}{\pi_s \partial M_{A,s}}$$

Setting all monetary stances to be equal, this condition becomes:

$$\mathsf{m}_{i,s} = \frac{2\alpha \mathsf{k}_{A,s} + (1-\alpha)\,\gamma_A^{B,\text{ cur } A} \mathsf{k}_{B,s} + (1-\alpha)\,\gamma_A^{C,\text{ cur } A} \mathsf{k}_{C,s}}{2\alpha + (1-\alpha)\,\gamma_A^{B,\text{ cur } A} + (1-\alpha)\,\gamma_A^{C,\text{ cur } A}} \qquad i = A,\, B,\, C$$

Under PCP-SYM monetary policy reacts only to shocks in the center: $\mathsf{m}_{i,s} = \mathsf{k}_{A,s}$. Under LCP-SYM or any of the DOL- cases, the monetary stances in each country is equal to (38), that is the policy rule of the center in a decentralized setting:

$$\mathsf{m}_{i,s} = \alpha \mathsf{k}_{A,s} + (1-\alpha)\,\frac{\mathsf{k}_{B,s} + \mathsf{k}_{C,s}}{2} \tag{42}$$

Therefore, the adoption of a peg by the periphery does not affect the policy choice of the center country, and only entails a loss of flexibility for the periphery countries. As a result, it cannot lead to a better outcome for periphery countries.

The solution was derived under the assumption that the monetary authorities in the center country sets their policy rule taking the monetary stances in the periphery countries as given. An alternative setting is to consider the center country as a strategic leader which internalizes the fact that the periphery countries peg their currencies. The center monetary authority then sets $M_B = M_C = M_A$ in the expression for consumption (29), and chooses its monetary stance to maximize $E \ln (C_A)$. We can show that the resulting monetary policy rule is still (42) regardless of the structure of invoicing.

4.4 Exchange rate volatility

Under PCP-SYM we write:

$$\begin{aligned} Var(\mathsf{s}_B) &= Var[\mathsf{m}_A - \mathsf{m}_B] = Var[\mathsf{k}_A - \mathsf{k}_B] \\ Var(\mathsf{s}_C) &= Var[\mathsf{k}_A - \mathsf{k}_C] \\ Var(\mathsf{s}_B - \mathsf{s}_C) &= Var[\mathsf{k}_C - \mathsf{k}_B] \end{aligned}$$

Under LCP-SYM we have:

$$\begin{aligned} Var(\mathsf{s}_B) &= Var(\mathsf{s}_C) = (2\alpha - 1)^2 Var\left[\mathsf{k}_A - \frac{\mathsf{k}_B + \mathsf{k}_C}{2}\right] \\ Var(\mathsf{s}_B - \mathsf{s}_C) &= 0 \end{aligned}$$

Under DOL-PCP and DOL-DOL we have:

$$\begin{aligned} Var(\mathsf{s}_B) &= Var\left[\alpha\left(\mathsf{k}_A - \frac{\mathsf{k}_B + \mathsf{k}_C}{2}\right) - \frac{1}{2}(\mathsf{k}_B - \mathsf{k}_C)\right] \\ Var(\mathsf{s}_C) &= Var\left[\alpha\left(\mathsf{k}_A - \frac{\mathsf{k}_B + \mathsf{k}_C}{2}\right) + \frac{1}{2}(\mathsf{k}_B - \mathsf{k}_C)\right] \\ Var(\mathsf{s}_B - \mathsf{s}_C) &= Var[\mathsf{k}_C - \mathsf{k}_B] \end{aligned}$$

Under DOL-LCP we have:

$$\begin{aligned} Var(\mathsf{s}_B) &= Var(\mathsf{s}_C) = \alpha^2 Var\left[\mathsf{k}_A - \frac{\mathsf{k}_B + \mathsf{k}_C}{2}\right] \\ Var(\mathsf{s}_B - \mathsf{s}_C) &= 0 \end{aligned}$$

4.5 Welfare

We compute the welfare by combining the various rules expected log consumption (33)-(35). We present the results in terms of deviations from the welfare under flexible prices. For instance: $\hat{U}_B = U_B - \Phi$. Under PCP-SYM monetary policy fully undoes the nominal rigidity and the welfare is brought to the flexible price level:

$$\hat{U}_A = \hat{U}_B = \hat{U}_C = 0 \tag{43}$$

Under LCP-SYM we have:

$$\begin{aligned} \hat{U}_A &= -\frac{\alpha(1-\alpha)}{2} Var\left[\mathsf{k}_A - \frac{\mathsf{k}_B + \mathsf{k}_C}{2}\right] - \frac{1-\alpha}{8} Var[\mathsf{k}_B - \mathsf{k}_C] \\ \hat{U}_B &= \hat{U}_C = -\frac{\alpha(1-\alpha)}{2} Var\left[\mathsf{k}_A - \frac{\mathsf{k}_B + \mathsf{k}_C}{2}\right] - \frac{\alpha}{8} Var[\mathsf{k}_B - \mathsf{k}_C] \end{aligned}$$

Under any variant of the DOL- cases, the welfare in country A is driven only by its own monetary policy, and we get:

$$\hat{U}_A = -\frac{\alpha(1-\alpha)}{2} Var\left[\mathsf{k}_A - \frac{\mathsf{k}_B + \mathsf{k}_C}{2}\right] - \frac{1-\alpha}{8} Var\left[\mathsf{k}_B - \mathsf{k}_C\right]$$

Which shows that the welfare is identical to the LCP-SYM case.

Under DOL-PCP we get:

$$\hat{U}_B = \hat{U}_C = -\frac{(1-\alpha)^3}{2} Var\left[\mathsf{k}_A - \frac{\mathsf{k}_B + \mathsf{k}_C}{2}\right]$$

Under DOL-LCP we get:

$$\hat{U}_B = \hat{U}_C = -\frac{(1-\alpha)^3}{2} Var\left[\mathsf{k}_A - \frac{\mathsf{k}_B + \mathsf{k}_C}{2}\right] - \frac{\alpha}{8} Var\left[\mathsf{k}_B - \mathsf{k}_C\right]$$

Under DOL-DOL we get:

$$\begin{aligned}
\hat{U}_B &= -\left[\frac{(1-\alpha)^3}{2} + \frac{\alpha^3}{4}\right] Var\left[\mathsf{k}_A - \frac{\mathsf{k}_B + \mathsf{k}_C}{2}\right] - \frac{\alpha}{16} Var\left[\mathsf{k}_B - \mathsf{k}_C\right] \\
&\quad -\frac{\alpha^2}{4} Covar\left[\mathsf{k}_A - \frac{\mathsf{k}_B + \mathsf{k}_C}{2}\right]\left[\mathsf{k}_B - \mathsf{k}_C\right] \\
\hat{U}_C &= -\left[\frac{(1-\alpha)^3}{2} + \frac{\alpha^3}{4}\right] Var\left[\mathsf{k}_A - \frac{\mathsf{k}_B + \mathsf{k}_C}{2}\right] - \frac{\alpha}{16} Var\left[\mathsf{k}_B - \mathsf{k}_C\right] \\
&\quad +\frac{\alpha^2}{4} Covar\left[\mathsf{k}_A - \frac{\mathsf{k}_B + \mathsf{k}_C}{2}\right]\left[\mathsf{k}_B - \mathsf{k}_C\right]
\end{aligned}$$

5 Optimal monetary policy in a cooperative setting

5.1 Intra-periphery cooperation

A first case of cooperation is limited to the periphery. The monetary authority in country A cares only about local consumption, as in the decentralized allocation. By contrast, the periphery authorities care about average consumption in the periphery. The optimal policy stances then satisfy:

$$
\begin{aligned}
0 &= \frac{\partial E \ln (C_A)}{\pi_s \partial M_{A,s}} \\
0 &= \frac{\partial E [\ln (C_B) + \ln (C_C)]}{\pi_s \partial M_{B,s}} = \frac{\partial E [\ln (C_B) + \ln (C_C)]}{\pi_s \partial M_{C,s}}
\end{aligned}
$$

We can show that monetary policy is then set exactly as in the decentralized allocation in all five cases we consider. This implies that the periphery countries cannot achieve a better outcome when cooperating among each other.

5.2 Monetary rules under world cooperation

A second case of cooperation extends to all countries. In this setup, policy makers in all three countries set policy to maximize the average of welfare across the three countries. The optimal policy stances then satisfy:

$$
\begin{aligned}
0 &= \frac{\partial E \ln (C_A) + \frac{1}{2} \partial E [\ln (C_B) + \ln (C_C)]}{\pi_s \partial M_{A,s}} \\
0 &= \frac{\partial E \ln (C_A) + \frac{1}{2} \partial E [\ln (C_B) + \ln (C_C)]}{\pi_s \partial M_{B,s}} \\
0 &= \frac{\partial E \ln (C_A) + \frac{1}{2} \partial E [\ln (C_B) + \ln (C_C)]}{\pi_s \partial M_{C,s}}
\end{aligned}
$$

Under the symmetric cases of PCP-SYM and LCP-SYM, the cooperative allocation is identical to the decentralized outcome.[3] In terms of the DOL-models, the cooperative rules are identical to the decentralized rules for the periphery countries. By contrast, the rule for monetary policy in country A is affected. Under DOL-PCP and DOL-LCP we get:

$$
\mathsf{m}_{A,s} = \frac{1}{2-\alpha} \mathsf{k}_{A,s} + \left(1 - \frac{1}{2-\alpha}\right) \frac{\mathsf{k}_{B,s} + \mathsf{k}_{C,s}}{2} \tag{44}
$$

while under DOL-DOL we get:

$$
\mathsf{m}_{A,s} = \frac{2}{4-\alpha} \mathsf{k}_{A,s} + \left(1 - \frac{2}{4-\alpha}\right) \frac{\mathsf{k}_{B,s} + \mathsf{k}_{C,s}}{2} \tag{45}
$$

[3] It is possible that cooperation could be beneficial in intermediate cases.

Intuitively, the first order solution with respect to the monetary stance in country A is written in general as:

$$0 = \underbrace{\mathsf{m}_{A,s} - \left[\alpha \mathsf{k}_{A,s} + (1-\alpha)\frac{\mathsf{k}_{B,s} + \mathsf{k}_{C,s}}{2}\right]}_{\text{block 1}} + \underbrace{(1-\alpha)(\mathsf{m}_{A,s} - \mathsf{k}_{A,s})}_{\text{block 2}} + \underbrace{\frac{\alpha}{2}\left(\mathsf{m}_{A,s} - \frac{\mathsf{k}_{B,s} + \mathsf{k}_{C,s}}{2}\right)}_{\text{block 3}}$$

Under the decentralized allocation the monetary authorities in country A only care about minimizing the prices faced by the consumer in country A, and react to a weighted average of productivity shocks, with the weights reflecting the weights of the various goods in the consumption basket. This is captured by block 1. Under a cooperative setup where the international role of currency A is limited to trade involving country A, the authority is also concerned with reducing the price of goods of country A sold in the periphery, an aspect captured by block 2. If the international role of currency A also includes the intra-periphery trade flows, then the authority also react to periphery shocks in order to limit the inefficient exchange rate movements between country B and C, an aspect captured by block 3. Note that cooperation has no bearing on the reaction to money demand shocks.

5.3 Pegging to the center currency

As before, we also consider a policy where each periphery country pegs its exchange rate to the center:

$$\mathsf{m}_{i,s} = \mathsf{m}_{A,s} \qquad i = B, C$$

The center's monetary policy is still set according to the following condition:

$$0 = \frac{\partial E \ln(C_A) + \frac{1}{2}\partial E\left[\ln(C_B) + \ln(C_C)\right]}{\pi_s \partial M_{A,s}}$$

Setting all monetary stances to be equal, this condition becomes:

$$\begin{aligned}
0 &= \left[\alpha + \frac{1-\alpha}{2}\gamma_B^{A,\text{ cur }A} + \frac{1-\alpha}{2}\gamma_C^{A,\text{ cur }A}\right](\mathsf{m}_{i,s} - \mathsf{k}_{A,s}) \\
&+ \left[\frac{1-\alpha}{2}\gamma_A^{B,\text{ cur }A} + \frac{\alpha}{4}\gamma_C^{B,\text{ cur }A}\right](\mathsf{m}_{i,s} - \mathsf{k}_{B,s}) \\
&+ \left[\frac{1-\alpha}{2}\gamma_A^{C,\text{ cur }A} + \frac{\alpha}{4}\gamma_B^{C\text{ cur }A}\right](\mathsf{m}_{i,s} - \mathsf{k}_{C,s})
\end{aligned}$$

for $i = A,\ B,\ C$.

In symmetric cases (PCP-SYM and LCP-SYM), all countries adopt the monetary stance that the center chooses in a decentralized setup, namely $\mathsf{m}_{i,s} = \mathsf{k}_{A,s}$ and (37). Under the DOL- cases all countries adopt a monetary rule that is exactly the one chosen by the center country in a cooperative allocation, namely (44) and (45). Pegging the exchange rate is then not optimal as it has no consequences for the policy rule of the center and limits the flexibility of policy in the periphery.

Another case would be a policy of peg and delegation, where the periphery countries peg their exchange rates and the center monetary authority sets its policy taking the peg into account, and maximizes the world welfare. Under this arrangement all monetary stances follow worldwide productivity:

$$\mathsf{m}_{i,s} = \mathsf{k}_{W,s}$$

5.4 Exchange rate volatility

Under DOL-PCP we get:

$$\begin{aligned}
Var(\mathsf{s}_B) &= Var\left[\frac{1}{2-\alpha}\left(\mathsf{k}_A - \frac{\mathsf{k}_B + \mathsf{k}_C}{2}\right) - \frac{1}{2}(\mathsf{k}_B - \mathsf{k}_C)\right] \\
Var(\mathsf{s}_C) &= Var\left[\frac{1}{2-\alpha}\left(\mathsf{k}_A - \frac{\mathsf{k}_B + \mathsf{k}_C}{2}\right) + \frac{1}{2}(\mathsf{k}_B - \mathsf{k}_C)\right] \\
Var(\mathsf{s}_B - \mathsf{s}_C) &= Var[\mathsf{k}_C - \mathsf{k}_B]
\end{aligned}$$

Under DOL-LCP we have:

$$\begin{aligned}
Var(\mathsf{s}_B) &= Var(\mathsf{s}_C) = \frac{1}{(2-\alpha)^2} Var\left[\mathsf{k}_A - \frac{\mathsf{k}_B + \mathsf{k}_C}{2}\right] \\
Var(\mathsf{s}_B - \mathsf{s}_C) &= 0
\end{aligned}$$

Under DOL-DOL we have:

$$
\begin{aligned}
Var\left(\mathsf{s}_B\right) &= Var\left[\frac{2}{4-\alpha}\left(\mathsf{k}_A-\frac{\mathsf{k}_B+\mathsf{k}_C}{2}\right)-\frac{1}{2}\left(\mathsf{k}_B-\mathsf{k}_C\right)\right] \\
Var\left(\mathsf{s}_C\right) &= Var\left[\frac{2}{4-\alpha}\left(\mathsf{k}_A-\frac{\mathsf{k}_B+\mathsf{k}_C}{2}\right)+\frac{1}{2}\left(\mathsf{k}_B-\mathsf{k}_C\right)\right] \\
Var\left(\mathsf{s}_B-\mathsf{s}_C\right) &= Var\left[\mathsf{k}_C-\mathsf{k}_B\right]
\end{aligned}
$$

5.5 Welfare

The welfare under PCP-SYM and LCP-SYM are the same as under the decentralized setting. Under DOL-PCP we have:

$$
\begin{aligned}
\hat{U}_A &= -\left[\frac{\alpha}{2}\left(\frac{1-\alpha}{2-\alpha}\right)^2+\frac{1-\alpha}{2}\left(\frac{1}{2-\alpha}\right)^2\right]Var\left[\mathsf{k}_A-\frac{\mathsf{k}_B+\mathsf{k}_C}{2}\right] \\
&\quad -\frac{1-\alpha}{8}Var\left[\mathsf{k}_B-\mathsf{k}_C\right] \\
\hat{U}_B &= \hat{U}_C=-\frac{1-\alpha}{2}\left(\frac{1-\alpha}{2-\alpha}\right)^2 Var\left[\mathsf{k}_A-\frac{\mathsf{k}_B+\mathsf{k}_C}{2}\right]
\end{aligned}
$$

Under DOL–LCP we have:

$$
\begin{aligned}
\hat{U}_A &= -\left[\frac{\alpha}{2}\left(\frac{1-\alpha}{2-\alpha}\right)^2+\frac{1-\alpha}{2}\left(\frac{1}{2-\alpha}\right)^2\right]Var\left[\mathsf{k}_A-\frac{\mathsf{k}_B+\mathsf{k}_C}{2}\right] \\
&\quad -\frac{1-\alpha}{8}Var\left[\mathsf{k}_B-\mathsf{k}_C\right] \\
\hat{U}_B &= \hat{U}_C=-\frac{1-\alpha}{2}\left(\frac{1-\alpha}{2-\alpha}\right)^2 Var\left[\mathsf{k}_A-\frac{\mathsf{k}_B+\mathsf{k}_C}{2}\right]-\frac{\alpha}{8}Var\left[\mathsf{k}_B-\mathsf{k}_C\right]
\end{aligned}
$$

Under DOL-DOL we have:

$$
\begin{aligned}
\hat{U}_A &= -\left[\frac{\alpha}{2}\left(\frac{2-\alpha}{4-\alpha}\right)^2 + \frac{1-\alpha}{2}\left(\frac{2}{4-\alpha}\right)^2\right] Var\left[\mathsf{k}_A - \frac{\mathsf{k}_B+\mathsf{k}_C}{2}\right] \\
&\quad -\frac{1-\alpha}{8} Var\left[\mathsf{k}_B - \mathsf{k}_C\right] \\
\hat{U}_B &= -\left[\frac{1-\alpha}{2}\left(\frac{2-\alpha}{4-\alpha}\right)^2 + \frac{\alpha}{4}\left(\frac{2}{4-\alpha}\right)^2\right] Var\left[\mathsf{k}_A - \frac{\mathsf{k}_B+\mathsf{k}_C}{2}\right] \\
&\quad -\frac{\alpha}{16} Var\left[\mathsf{k}_B - \mathsf{k}_C\right] - \frac{\alpha}{2}\frac{1}{4-\alpha} Covar\left[\mathsf{k}_A - \frac{\mathsf{k}_B+\mathsf{k}_C}{2}\right]\left[\mathsf{k}_B - \mathsf{k}_C\right] \\
\hat{U}_C &= -\left[\frac{1-\alpha}{2}\left(\frac{2-\alpha}{4-\alpha}\right)^2 + \frac{\alpha}{4}\left(\frac{2}{4-\alpha}\right)^2\right] Var\left[\mathsf{k}_A - \frac{\mathsf{k}_B+\mathsf{k}_C}{2}\right] \\
&\quad -\frac{\alpha}{16} Var\left[\mathsf{k}_B - \mathsf{k}_C\right] + \frac{\alpha}{2}\frac{1}{4-\alpha} Covar\left[\mathsf{k}_A - \frac{\mathsf{k}_B+\mathsf{k}_C}{2}\right]\left[\mathsf{k}_B - \mathsf{k}_C\right]
\end{aligned}
$$

We next turn to the cooperative peg where the monetary stance in all countries track the worldwide productivity shocks. Regardless of the invoicing structure, (33)-(35) imply:

$$
\begin{aligned}
\hat{U}_A &= -\frac{1}{8} Var\left[\mathsf{k}_A - \frac{\mathsf{k}_B+\mathsf{k}_C}{2}\right] - \frac{1-\alpha}{8} Var\left[\mathsf{k}_B - \mathsf{k}_C\right] \\
\hat{U}_B &= \hat{U}_C = -\frac{1}{8} Var\left[\mathsf{k}_A - \frac{\mathsf{k}_B+\mathsf{k}_C}{2}\right] - \frac{\alpha}{8} Var\left[\mathsf{k}_B - \mathsf{k}_C\right]
\end{aligned}
$$

6 The case of periphery-wide shocks

6.1 Exchange rate volatility

Our main results are highlighted by considering the case where shocks in countries B and C are always identical ($\mathsf{k}_B = \mathsf{k}_C$), so the world is driven by center and periphery shocks.

When monetary policy is conducted in a decentralized fashion, the exchange rate is always most volatile in the PCP-SYM case and least volatile in the LCP-SYM case. Its volatility under any DOL- case falls in between:

$$\begin{aligned} Var\left(\mathsf{s}_B\right)_{\text{PCP-SYM, decentralized}} &= Var\left[\mathsf{k}_A - \mathsf{k}_B\right] \\ Var\left(\mathsf{s}_B\right)_{\text{LCP-SYM, decentralized}} &= \left(2\alpha - 1\right)^2 Var\left[\mathsf{k}_A - \mathsf{k}_B\right] \\ Var\left(\mathsf{s}_B\right)_{\text{DOL-, decentralized}} &= \alpha^2 Var\left[\mathsf{k}_A - \mathsf{k}_B\right] \end{aligned}$$

where:

$$1 \geq \alpha^2 \geq \left(2\alpha - 1\right)^2$$

with equality when $\alpha = 1$.

When monetary policy is conducted in a cooperative fashion under a DOL- case, the exchange rate is least volatile under the DOL-DOL case:

$$\begin{aligned} Var\left(\mathsf{s}_B\right)_{\text{DOL-PCP/LCP, cooperative}} &= \left(\frac{1}{2-\alpha}\right)^2 Var\left[\mathsf{k}_A - \mathsf{k}_B\right] \\ Var\left(\mathsf{s}_B\right)_{\text{DOL-DOL, cooperative}} &= \left(\frac{2}{4-\alpha}\right)^2 Var\left[\mathsf{k}_A - \mathsf{k}_B\right] \end{aligned}$$

where:

$$1 \geq \left(\frac{1}{2-\alpha}\right)^2 > \left(\frac{2}{4-\alpha}\right)^2$$

Comparing the decentralized and cooperative outcome, the exchange rate is more volatile under a cooperative setup in the DOL-PCP and DOL-LCP cases, as well as under the DOL-DOL case if the countries are relatively integrated:

$$\begin{aligned} \left(\frac{1}{2-\alpha}\right)^2 &> \alpha^2 \\ \left(\frac{2}{4-\alpha}\right)^2 &> \alpha^2 \Leftrightarrow \alpha < 0.58 \end{aligned}$$

6.2 Welfare

Under a decentralized setup, the welfare of country A is always (except in PCP-SYM):

$$\hat{U}_{A,\text{ decentralized}} = -\left(1 - \alpha\right) \frac{\alpha}{2} Var\left[\mathsf{k}_A - \mathsf{k}_B\right]$$

The welfare of the periphery countries is:

$$\begin{aligned}
\hat{U}_{B,\text{ LCP-SYM, decentralized}} &= -(1-\alpha)\frac{\alpha}{2}Var\left[\mathsf{k}_A-\mathsf{k}_B\right] \\
\hat{U}_{B,\text{ DOL–PCP/LCP, decentralized}} &= -\frac{(1-\alpha)^3}{2}Var\left[\mathsf{k}_A-\mathsf{k}_B\right] \\
\hat{U}_{B,\text{ DOL-DOL, decentralized}} &= -\left[\frac{(1-\alpha)^3}{2}+\frac{\alpha^3}{4}\right]Var\left[\mathsf{k}_A-\mathsf{k}_B\right]
\end{aligned}$$

Focusing on the DOL- cases for brevity, an exchange rate peg entails no cost for country A, but reduces the welfare of the periphery countries.

Under a cooperative allocation, the welfare levels are different under the DOL- cases. Under DOL-PCP and DOL-LCP we have:

$$\begin{aligned}
\hat{U}_{A,\text{ DOL–PCP/LCP, cooperative}} &= -\frac{1-\alpha}{2}\left(\frac{1}{2-\alpha}\right)^2\left[\alpha(1-\alpha)+1\right]Var\left[\mathsf{k}_A-\mathsf{k}_B\right] \\
\hat{U}_{B,\text{ DOL–PCP/LCP, cooperative}} &= -\frac{1-\alpha}{2}\left(\frac{1-\alpha}{2-\alpha}\right)^2Var\left[\mathsf{k}_A-\mathsf{k}_B\right]
\end{aligned}$$

Under DOL-DOL case we have:

$$\begin{aligned}
\hat{U}_{A,\text{ DOL–DOL, cooperative}} &= -\left[\frac{\alpha}{2}\left(\frac{2-\alpha}{4-\alpha}\right)^2+\frac{1-\alpha}{2}\left(\frac{2}{4-\alpha}\right)^2\right]Var\left[\mathsf{k}_A-\mathsf{k}_B\right] \\
\hat{U}_{B,\text{ DOL–DOL, cooperative}} &= -\left[\frac{1-\alpha}{2}\left(\frac{2-\alpha}{4-\alpha}\right)^2+\frac{\alpha}{4}\left(\frac{2}{4-\alpha}\right)^2\right]Var\left[\mathsf{k}_A-\mathsf{k}_B\right]
\end{aligned}$$