Credit Spreads, Financial Crises, and Macroprudential Policy

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Abstract

Credit spreads display occasional spikes and are more strongly countercyclical in times of
financial stress. Financial crises are extreme cases of this nonlinear behavior, featuring
skyrocketing credit spreads, sharp losses in bank equity, and deep recessions. We develop a
macroeconomic model with a banking sector in which banks' leverage constraints are
occasionally binding and equity issuance is endogenous. The model captures the nonlinearities in
the data and produces quantitatively realistic crises. Precautionary equity issuance makes crises
infrequent but does not prevent them altogether. When determining the intensity of capital
requirements, the macroprudential authority faces a trade-off between the benefits of reducing the
risk of a financial crisis and the welfare losses associated with banks' constrained ability to
finance risky capital investments.

Key words: financial intermediation, sudden stops, leverage constraints, occasionally
binding constraints, financial stability policy

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1   Introduction

The recent wave of financial crises across the globe has put financial stability risks, and the potential role of "macroprudential" policies in addressing them, at the forefront of policy discussions. At the same time, it has renewed interest in macroeconomic models that can adequately capture financial crises. In this paper, based on data from several European countries and from the U.S., we characterize three stylized facts about financial stress. First, credit spreads (defined as average yields of corporate bonds relative to government bonds of similar maturity) display occasional spikes. Second, the relationship between credit spreads and real activity is highly asymmetric: when credit spreads are elevated, their correlation with GDP is much stronger than when they are subdued. Taken together, the first two facts are suggestive of strong nonlinearities in the relation between financial stress and the real economy. Finally, we show that banking crisis episodes feature an extreme form of these nonlinearities, evidenced by skyrocketing credit spreads, sharp losses in banking sector equity, and deep recessions.[1]

Recent quantitative nonlinear macroeconomic models like Mendoza (2010) or Bianchi and Mendoza (forthcoming) explain deep recessions during sudden stops as a result of the amplification induced by a collateral constraint imposed (exogenously) at the country level. Because they do not explicitly model financial intermediaries, however, these frameworks cannot account for the disruptions in banking sectors that are typically at the heart of banking crises,[2] nor can they address the effects of macroprudential policies that impose constraints on banks' leverage. On the other hand, quantitative macroeconomic models with banking sectors (for example, Gertler and Kiyotaki (2010) or Gertler and Karadi (2011)) typically generate banking crises by relying on large, unexpected shocks to the banking sector, and thus cannot account for occasional severe financial distress as suggested by the stylized facts just described.

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[1]Several authors have also emphasized the notion that banking crises are events characterized by strong nonlinearities—for example, Merton (2009), Kenny and Morgan (2011), Hubrich et al. (2013), or He and Krishnamurthy (2014).

[2]Such disruptions are exemplified, as we show, by the enormous losses in financial-sector equity that occur during a typical banking crisis episode. Sudden stops, on the other hand, tend to feature very small equity losses, on average, in the banking sector. Much more important for the latter type of events are disruptions in the supply of external financing, as argued for example by Claessens and Kose (2013). This makes frameworks based on Mendoza (2010) appealing to capture sudden stop episodes. At the same time, the distinct features of banking crises point to the need of an alternative approach to properly account for this type of episodes.

Accordingly, this paper attempts to fill this gap by developing a macroeconomic model with financial intermediaries (banks, for short), which can account for the strong nonlinearities documented earlier. After establishing that the model can account for the facts, we use it to examine the desirability of macroprudential policy measures directed at strengthening bank balance sheets. In the model, we find, macroprudential policy can be effective in reducing the risk of banking crises, thereby leading to important welfare gains.

In particular, we propose a dynamic stochastic small open economy model in which banks' leverage constraints are occasionally binding and equity issuance is endogenous.[3,4] Banks in our model are unconstrained most of the time, which accounts for relatively low levels of credit spreads in normal times. Because banks anticipate future shocks may put them against their leverage constraint, they issue equity at a positive rate in these times, contributing to a stronger net worth position and thereby reducing the probability of a financial crisis. As a consequence, the economy spends most of the time in the unconstrained region, in which it exhibits fluctuations similar to those of a standard neoclassical model, and which features subdued credit spreads and relatively high net worth and asset prices.

Nonetheless, financial crises are inevitable, as in the data. They emerge infrequently in our model as a consequence of the nonlinearity induced by the leverage constraint, which binds when aggregate bank net worth is sufficiently low. When the constraint binds, the economy enters financial crisis mode. Low bank net worth raises credit spreads sharply due to banks' inability to extend financing to the private sector, which in turn slows the economy, depressing asset prices and bank net worth further.[5] In addition to financial frictions affecting banks' financing of investment expenditures, our model also features working capital frictions that hinder banks' ability to lend

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[3]The reason for using a small open economy framework is twofold: First, most of the countries in our sample are better characterized as small open developed economies. Second, our purpose in this paper is to offer a general macroeconomic framework in which we can compare the policy implications of several prudential tools, such as bank capital requirements and capital controls.

[4]As in Gertler and Kiyotaki (2010), Gertler and Karadi (2011) and Gertler, Kiyotaki and Queralto (2012), in our model an agency friction in the short-term debt market may limit banks' leverage. Unlike these papers, in our framework the constraint is occasionally binding.

[5]This is the well known financial accelerator mechanism. See, for example, Bernanke, Gertler and Gilchrist (1999), Kiyotaki and Moore (1997), Jermann and Quadrini (2012) and Christiano, Motto and Rostagno (2014) for models that introduce this mechanism in macroeconomic frameworks. Unlike these papers, our global solution technique allows us to capture the nonlinear nature of the financial accelerator.

to nonfinancial firms to pay for the wage bill. The latter type of friction gives rise to contemporaneous declines in hours and output in times of elevated credit spreads (which in turn also feed back into financial conditions), as greater credit costs work to increase the effective cost of labor. This helps the model account for the sharp output downturns observed in the data.

We undertake a quantitative analysis of the model economy calibrated on data from our sample. The model is buffeted by exogenous stochastic disturbances to total factor productivity, the country interest rate, and the quality of financial sector assets. We find that the model does well in accounting for the aforementioned three empirical regularities. The economy endogenously switches between normal times (featuring low credit spreads) and occasional financial crises (when credit spreads rise sharply). In this way, the model can generate the long right tail in the distribution of credit spreads. Our model also captures well the asymmetry in the relation between credit spreads and economic activity. A binding leverage constraint gives rise to an amplification mechanism via the financial accelerator, thereby strengthening the link between credit spreads and the real economy.

Finally, our model generates banking crisis episodes that are quantitatively consistent with the evidence. In particular, crisis periods feature severe disruption in financial intermediation, exemplified by large increases in credit spreads and sharp losses in bank equity, as well as plunges in domestic investment and output, with magnitudes consistent with the data. Crises in the model are triggered not by unusually large shocks but by moderately adverse sequences of all three disturbances, which push the economy toward the constrained region and eventually trigger the constraint.

Having demonstrated that the model does a good job of accounting for the facts, we next use it to assess the desirability of macroprudential policy directed at enhancing financial stability. Within our framework, when the constraint binds, banks' ability to borrow is affected by asset prices, since the latter affect net worth. This may introduce a pecuniary externality in banks' choice of equity issuance: when the constraint binds, a better-capitalized bank balance sheet position would work to contain the decline in asset prices, thereby improving aggregate net worth—an effect not internalized by atomistic banks when choosing how much equity to issue *ex ante*. The existence of a pecuniary externality creates a rationale for macroprudential policy.

We consider two types of macroprudential policies. The first type, which we call

"government subsidy," consists of a subsidy on banks' equity issuance (financed via lump-sum taxes on households) which tilts banks' incentives in favor of raising more equity. This policy has the advantage of clearly illustrating the extent to which banks' privately optimal choice of equity issuance is socially inefficient. As we show, the degree of inefficiency is high, and the welfare gains from an appropriately chosen subsidy are large. This type of policy, however, might be hard to implement in practice. For example, the government might not have access to lump-sum taxes, or political economy considerations might make it infeasible to subsidize banks.

Accordingly, the second type of policy we consider consists of a regulatory constraint on banks' leverage. Unlike the government subsidy, this policy has the advantage of closely reproducing real-world policies such as the capital requirements implemented within the Basel framework. Our simulations suggest that the regulatory policy may also be welfare improving, although the welfare gain is in general smaller than the gain from the government subsidy. If the regulatory limit is very tight, it leads to welfare losses, because the gains due to reduced frequency of financial crises are not enough to compensate the losses due to lower levels of physical capital stock (which arise because the policy hinders banks' ability to finance risky capital investments). Hence, in determining the intensity of capital requirements the regulatory agency has to evaluate the tradeoff between constraining banks' lending and the probability of a financial crisis.

This paper is related to several strands in the literature. As mentioned before, the model economy proposed in this paper endogenously switches between normal times and financial crisis times, as in the recent Nonlinear Dynamic Stochastic General Equilibrium (NDSGE) models developed by Bianchi (2010), Mendoza (2010), and others.[6] However, in our model, borrowing constraints arise endogenously as a result of an explicit agency friction affecting banks, as in Gertler and Karadi (2011), Gertler and Kiyotaki (2010) and related work. This is in contrast with the NDSGE literature, which imposes exogenous collateral constraints to capture sudden stop dynamics.

In addition, our model suggests different policy prescriptions than those emphasized by models of sudden stops. Macroprudential policies considered in the NDSGE literature mainly focus on preventing overborrowing in international financial markets (via pigovian taxation) to try to prevent sudden stop risks. Our analysis, instead,

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[6]See also Benigno, Chen, Otrok, Rebucci and Young (2012), Schmitt-Grohe and Uribe (2016) and Bianchi and Mendoza (forthcoming).

highlights policies directed at improving the net worth of the domestic banking sector. Existing models of sudden stops, even those with some form of financial sector (for example, Brunnermeier and Sannikov (2015)), cannot address the question of the relative desirability of capital controls and domestic macroprudential policies as the implications of these two policy tools are hard to differentiate in these models.[7]

Novel features of our setup relative to Gertler and Karadi (2011) and Gertler and Kiyotaki (2010), on the other hand, are twofold. First, these papers analyze the model's local behavior around a steady state in which the constraint always binds. We instead focus on the global implications when the constraint binds only occasionally. This feature allows us to study financial crisis dynamics, which occur far from the steady state, without having to resort to unrealistically large shocks. Second, we allow banks to raise new equity. In Gertler and Kiyotaki (2010) and related frameworks, banks' net worth typically only reflects the mechanical evolution of retained earnings, and therefore any explicit precautionary behavior by banks is ruled out by assumption. By allowing this new choice margin for banks, then, we can analyze whether government policies may improve on *laissez-faire* by manipulating that margin.[8],[9]

In a recent work, Boissay, Collard and Smets (2016) also augment a DSGE framework with a banking sector to account for nonlinearities, with a particular focus on generating boom-bust dynamics. Our framework differs from theirs in several important ways, particularly concerning the modeling of the banking sector. In Boissay et al. (2016), banks are risk neutral and one-period-lived. By contrast, within our frame-

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[7]Akinci and Olmstead-Rumsey (2015) document that domestic macroprudential policies, especially capital requirements, have been used actively in small open developed economies after the global financial crisis. They also document that the use of macroprudential measures increased after the crisis in emerging economies as well, while countries in this group have also implemented capital controls. However, still very little is known on whether domestic macroprudential policies will ultimately be effective in preventing future financial crises, and on how effective these measures are *vis-à-vis* capital controls. We provide an encompassing quantitative framework to address these questions.

[8]In Gertler, Kiyotaki and Queralto (2012) banks are allowed to issue outside equity, which has hedging value for them. By contrast, here we allow banks to raise inside equity. In addition, Gertler, Kiyotaki and Queralto (2012) continue to perform local analysis around a steady state in which the constraint binds. In our framework, instead, the constraint is occasionally binding, and the likelihood that the constraint binds in the future is a key determinant of banks' equity issuance.

[9]A recent paper by Bocola (2016) also allows banks' constraint to be occasionally binding, but focuses on the transmission of sovereign risk to the real sector through banks' balance sheets. Instead, our focus is on capturing occasional financial crises, and on showing the importance of banks' risk-taking behavior in driving the likelihood of future crises.

work banks are risk-averse and have infinite horizons. The latter feature is critical for the model to capture how fluctuations in the value of banks' assets, together with leveraged lending, contribute to volatility in banking-sector net worth—an important aspect of the interaction between financial stress and the real economy. Further, such asset-price effects—which are absent in Boissay et al. (2016)—might in fact work to mitigate boom-bust dynamics: bad news about the future typically depress asset prices (and therefore net worth) today, making it more likely that the crisis occurs today rather than in the future. One final difference is that our framework, unlike Boissay et al. (2016), explicitly models banks' financial structure, thereby allowing analysis of capital requirements and other macroprudential policies.

Other related papers are Cespedes et al. (2016), Brunnermeier and Sannikov (2014), and He and Krishnamurthy (2014). Unlike these papers, our focus is to offer a framework that does not stray too far from the standard quantitative DSGE models used in policy analysis, and that is tractable enough to accommodate the features that are present in that literature.

The remainder of the paper is organized as follows. Section 2 documents stylized facts on credit spreads and financial crises. Section 3 presents model. The quantitative analysis is conducted in sections 4 and 5. Section 4 describes the model's functional forms and calibration. Section 5 analyzes the quantitative behavior of the model economy in both normal times and financial crisis times, and also explores the characteristics of the financial crisis episodes produced by the model. Section 6 analyzes macroprudential policy. Section 7 concludes.

2   Facts on Credit Spreads and Financial Crises

We use quarterly data from several euro area countries, the UK, and the US. The period is 1999-2015 for the euro area countries, 1983-2015 for the UK, and 1973-2011 for the US. This choice of sample is governed by the availability of comprehensive private-sector measures of credit spreads—a key variable in our theoretical analysis. We document three facts on credit spreads and financial crises against which we will judge our modeling framework.

Our first fact is that credit spreads display occasional very large spikes. This regularity is evident in Figure 1, showing the histogram of credit spreads for the countries in our sample: the empirical distribution of spreads clearly has a fat right

Figure 1: Histogram of Credit Spreads
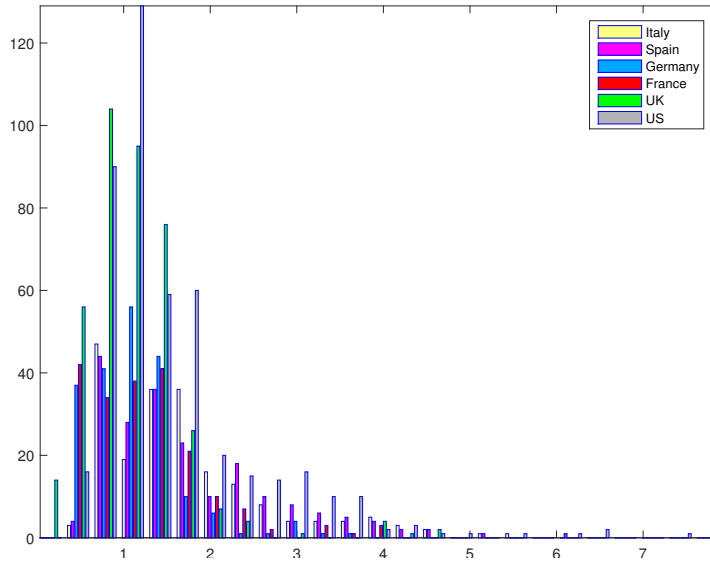
Note: Credit spreads stand for corporate credit spreads for non-financial firms. They are calculated as the average spreads between the yield of private-sector bonds in Italy, Spain, Germany and France relative to the yield on German government securities, in the UK relative to UK government securities, and in the US relative to US government securities, of matched maturities. Data sources: Gilchrist and Mojon (2014), Bank of England, Gilchrist and Zakrajsek (2012).

tail. Spreads tend to hover around 100 basis points a large fraction of the time, while they infrequently take values as large as 700 basis points. This visual impression is confirmed by the skewness and kurtosis coefficients of the distribution of credit spreads, shown in the first two rows of Table 1. The positive skew and the excess kurtosis (recall that a normal distribution has zero skewness and a kurtosis of 3) indicate that the distribution of credit spreads is asymmetric to the right and has heavy tails. A Jarque-Bera test, as adapted by Bai and Ng (2005) to allow for serially correlated data, comfortably rejects the null of normality.[10]

We next turn to the link between credit spreads and economic activity. Here our second fact is that the relationship between credit spreads and activity is *also* highly asymmetric: as shown in Figure 2, the correlation between credit spreads and real GDP when the former are elevated relative to their mean (right panel) is much

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[10]There is also evidence in favor of rejecting zero excess skewness and kurtosis separately, albeit only at 10 percent confidence in the case of the latter—an unsurprising finding given the low power of individual kurtosis tests, as Bai and Ng (2005) emphasize.

Table 1: Credit Spreads, Summary Statistics

	Coefficient	P-value
Skewness	2.235	0.0179
Kurtosis	10.793	0.0759
Jarque-Bera	17.094	9.7×10^{-5}
ρ^-	-0.130	0.0104
ρ^+	-0.427	2.3×10^{-11}
$\rho^+ - \rho^-$	-0.297	2.6×10^{-6}

Note: Left column shows the statistic, and right column the corresponding (one-sided) p-value. P-values for skewness, kurtosis and Jarque-Bera statistic calculated following Bai and Ng (2005), who adapt the Jarque-Bera tests of normality (Jarque and Bera (1980)) to allow for serially correlated data. ρ^- and ρ^+ denote, respectively, the correlation between spreads and GDP for negative and for positive spread deviations from the mean.

Figure 2: Credit Spreads and Output
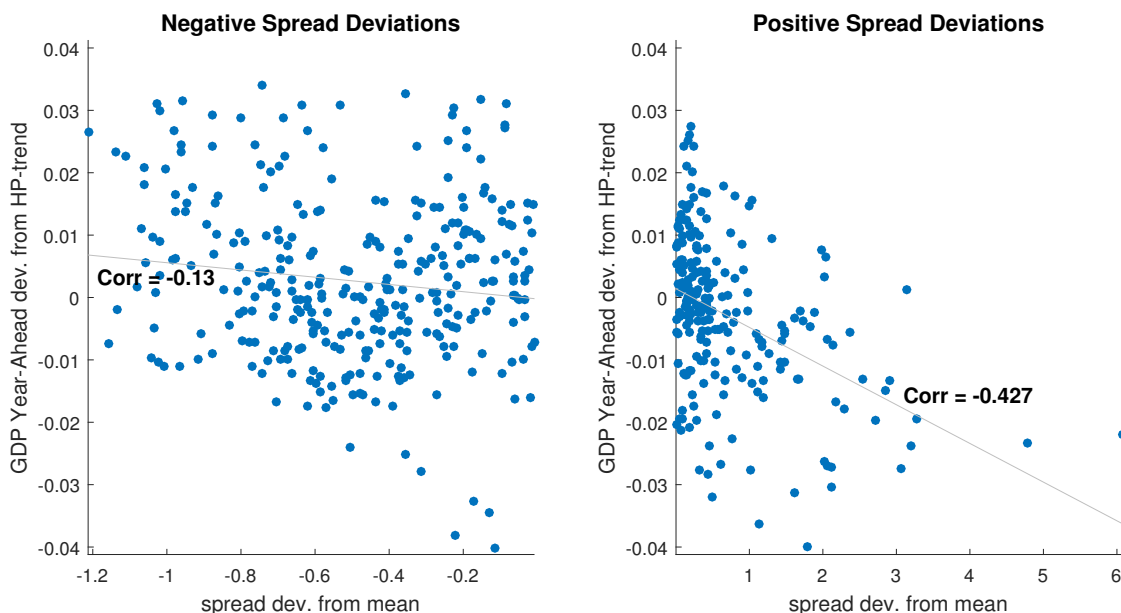
Note: The left (right) panel shows the relationship between year-ahead real GDP, expressed as a deviation from its HP trend, and the negative (positive) deviations of the credit spread from its mean for the countries in our sample (Italy, Spain, Germany, France, UK and the US).

stronger than when spreads are below the mean (left panel).[11] Put differently, credit spreads are not only countercyclical (a well-known fact), but the strength of their

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[11]Here we measure real GDP as year-ahead deviations from its HP trend.

Figure 3: Crises Event Study
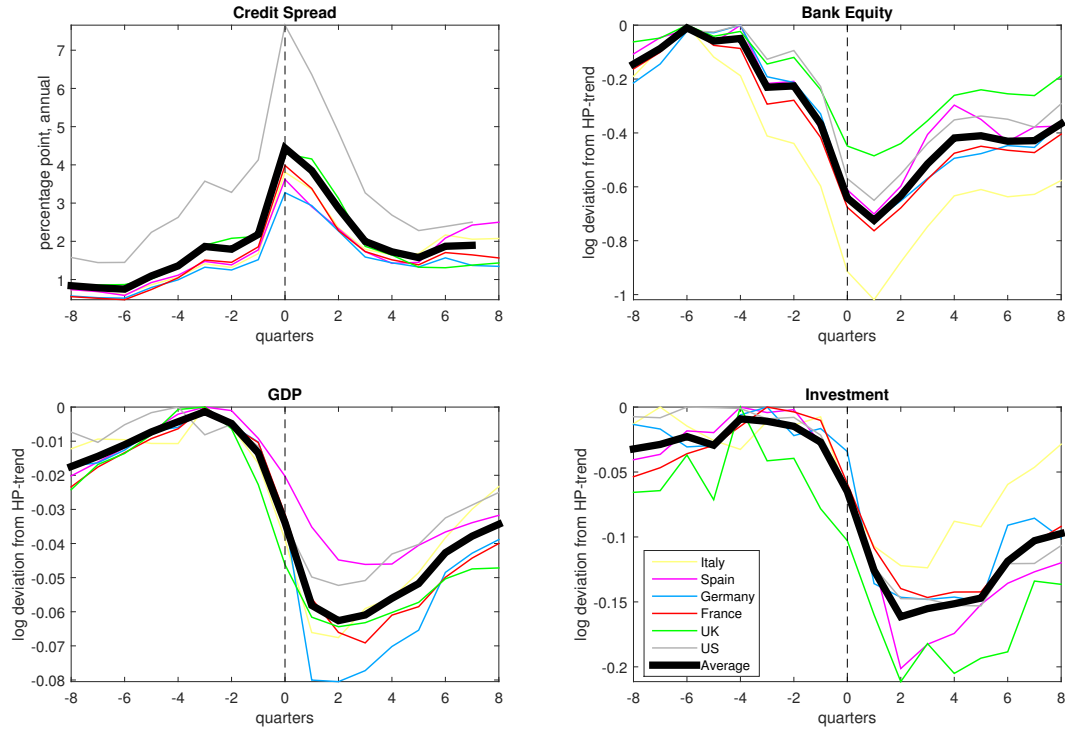
Note: Credit spreads are in percentage points. Bank equity, GDP and investment are all deflated by GDP deflator, and HP detrended with a smoothing parameter 1600. The events are centered at the quarter when the credit spreads peaked within the systemic banking crises episodes identified by Laeven and Valencia (2012) for each country. The events based on 6 systemic banking crises episodes, one in each country. The event window includes 8 quarters before and 8 quarters after the event, and all series but credit spreads are normalized at their respective pre-crisis peaks.

countercyclicality tends to be higher when they are relatively elevated. The fourth and fifth rows of Table 1 (in which ρ^- and ρ^+ denote, respectively, the correlation between spreads and GDP for negative and for positive spread deviations from the mean) provide tests for the hypothesis of zero correlation, which are both rejected at 5 percent significance level. The last row of the Table tests the hypothesis that $\rho^+ = \rho^-$ (against the alternative that $\rho^+ < \rho^-$): the hypothesis of equal correlations is also decidedly rejected.[12]

Our third fact examines the average behavior of macroeconomic aggregates around

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[12]Stein (2014) has emphasized a similar fact for the US economy, using the "excess bond premium" (which was proposed in Gilchrist and Zakrajsek (2012)) as measure of credit spread. The excess bond premium measures credit spreads net of expected default losses on corporate bonds.

financial crisis events, as identified by Laeven and Valencia (2012). We find that these episodes are associated with unusually elevated credit spreads and deep recessions, occurring along with sharp losses in banking sector equity. In Figure 3 we plot an event analysis of the financial crisis episodes in our sample. During financial crises, credit spreads rise sharply, reaching about 450 basis points at the peak on average, and the value of bank equity (as measured by financial sector equity indices) declines dramatically, by roughly 70 percent on average relative to trend.[13] These adverse developments on the financial side occur alongside sharp macroeconomic contractions, with GDP and investment falling about 4 and 7 percent below trend, respectively, in the quarter when the spread peaks. They then continue to fall and bottom out at about 6 and 16 percent below their respective trends two quarters later, before gradually moving back toward pre-crisis levels. Thus, these crisis episodes emerge as an extreme manifestation of the asymmetric and nonlinear behavior documented above: they coincide with unusually elevated levels of credit spreads, along with sharp macroeconomic downturns.[14]

In the next section, we develop a model featuring strong nonlinearities through financial-market frictions which can account for the facts documented above.

3   The Model

The core model is a small open economy extension of the macroeconomic model with banks presented in Gertler and Kiyotaki (2010) (abstracting from liquidity risks). Banks make risky loans to nonfinancial firms and collect deposits from both domestic households and foreigners. Because of an agency problem, banks may be constrained in their access to external funds.

We introduce three novel features to the model, all of which prove necessary for the model to generate empirically realistic dynamics of real and financial variables.

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[13]These drops in financial equity values are generally outsized relative to other sectors of the economy. For example, the S&P500 Financial Index collapsed by about 75 percent around 2008 (peak to trough), compared with a decline of 40 percent in the S&P500 Industrial Index.

[14]The event analysis above is based on six financial crises—one in each country in our baseline sample, for which we are constrained by the availability of credit spread data. However, in Appendix A, we document that the average behavior of real GDP, investment and banking-sector equity around financial crises in an extended sample including twenty-three crises during the 1985-2015 period is fairly similar to our baseline results. The event study figure for the extended sample along with the list of countries included, which were chosen based on availability of banking-sector equity data, are presented in Appendix A.

First, banks' constraints are not permanently binding, as in much of the related literature, but instead bind only occasionally. In normal, or "tranquil," times, banks' constraints are not binding: credit spreads are small and the economy's behavior is similar to a frictionless neoclassical framework. When the constraint binds the economy enters into financial crisis mode: credit spreads rise sharply and investment and credit collapse, consistent with the evidence.

The second novel feature of our setup is that banks are allowed to raise new equity from households, so that the evolution of bank net worth reflects banks' endogenously chosen rate of new equity issuance, as well as the mechanical accumulation of retained earnings. This equity issuance is precautionary and helps banks avoid a binding leverage constraint in the future.

Finally, nonfinancial firms are subject to working capital constraints. The novelty here is to show how one can combine agency frictions applied to the banking sector with frictions arising from the need for working capital loans.

3.1 Households

Each household is composed of a constant fraction $(1 - f)$ of workers and a fraction f of bankers. Workers supply labor to the firms and return their wages to the household. Each banker manages a financial intermediary ("bank") and similarly transfers any net earnings back to the household. Within the family there is perfect consumption insurance.

Households do not hold capital directly. Rather, they deposit funds in banks. The deposits held by each household are in intermediaries other than the one owned by the household. Bank deposits are riskless one-period securities. Consumption, C_t, bond holdings, B_t, and labor decisions, H_t, are given by maximizing the discounted expected future flow of utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, H_t), \tag{1}$$

subject to the budget constraint

$$C_t + B_t \leq W_t H_t + R_{t-1} B_{t-1} + \Pi_t \tag{2}$$

$\mathbb{E}_t$ denotes the mathematical expectation operator conditional on information available at time t, and $\beta \in (0,1)$ represents a subjective discount factor. The variable W_t is the real wage, R_t is the real interest rate received from holding one period bond, and Π_t is total profits distributed to households from their ownership of both banks and firms. The first order conditions of the household's problem are presented in Appendix B.

3.2 Banks

Banks are owned by the households and operated by the bankers within them. In addition to its own equity capital, a bank can obtain external funds from both domestic households, b_t, and foreign investors, b_t^*, such that total external financing available to the bank is given by $d_t = b_t + b_t^*$. We assume that both domestic deposits and foreign borrowing are one-period non-contingent debt. Thus, by arbitrage their returns need to be equalized in equilibrium, a condition we impose at the onset.[15]

In addition, banks in period t can raise an amount e_t of new equity. The new equity is available in the following period to make risky loans to nonfinancial firms, together with the equity accumulated via retained earnings and with any external borrowing. Accordingly, in each period the bank uses its net worth n_t (which includes equity raised in the previous period) and external funds d_t, to purchase securities issued by nonfinancial firms, s_t, at price Q_t. In turn, nonfinancial firms use the proceeds to finance their purchases of physical capital.

Banks also borrow external funds in the non-contingent debt market, $d_{W,t}$, to finance intraperiod working capital loans made to nonfinancial firms, $s_{W,t}$, such that $d_{W,t} = s_{W,t}$ in each period. Nonfinancial firms, in turn, use these funds to pay for a fraction of wage bill in advance of production.[16]

Agency Friction and Incentive Constraint

We assume that banks are "specialists" who are efficient at evaluating and monitoring nonfinancial firms and also at enforcing contractual obligations with these

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[15]Here we denote banks' individual variables with lowercase letters, and later use uppercase to refer to their aggregate counterparts.

[16]We follow the timing assumptions in Neumeyer and Perri (2005) for intraperiod working capital loans. See Appendix C for a complete description of banks' working capital loans, including details on our timing assumptions on the borrowing-lending relationship between banks and nonfinancial firms.

borrowers. That is why firms rely exclusively on banks to obtain funds, and the contracting between banks and nonfinancial firms is frictionless. However, as in Gertler and Kiyotaki (2010) and related papers, we introduce an agency problem whereby the banker managing the bank may decide to default on its obligations and instead transfer a fraction of assets to his family, in which case it is forced into bankruptcy and its creditors can recover the remaining funds. In recognition of this possibility, creditors potentially limit the funds they lend to banks. In our setup, banks may or may not be credit constrained, depending on whether or not they are perceived to have incentives to disregard their contractual obligations.

More specifically, after having borrowed external funds (both $d_{W,t}$ and d_t) but before repaying its creditors, the bank may decide to default on its obligations and divert fraction θ of risky loans and fraction ω of working capital loans. In this case, the bank is forced into bankruptcy and its creditors recover the remaining funds (of both risky loans and working capital loans). If the bank decides to honor the debt borrowed in the noncontingent debt market to finance working capital loans ($d_{W,t}$), it pays back within the same time period, when it receives the repayments from goods-producing firms (after production takes place). The net proceeds to the bank from working capital loans are given by $(R_{L,t} - R_{t-1})s_{W,t}$, where $R_{L,t}$ is the gross rate of return on working capital loans and R_{t-1} is the rate of return on one-period (risk-free) bonds held by the bank's creditors from $t-1$ to t.

To ensure that the bank does not divert funds, the incentive constraint must hold:

$$V_t \geq \theta \left[Q_t s_t - (R_{L,t} - R_{t-1})s_{W,t} \right] + \omega s_{W,t} \tag{3}$$

where V_t stands for the continuation value of the bank. This constraint requires that the bank's continuation value be higher than the value of the diverted funds.[17]

Equity Issuance

One of the novel features of our model is that banks are allowed to raise new equity each period from the households they belong to, provided that they survive into the following period. As is standard in the literature (see, for example, Gertler and Kiyotaki (2010) or Gertler and Karadi (2011)), we assume that only with *i.i.d.*

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[17]The term $(R_{L,t} - R_{t-1})s_{W,t}$ in the right-hand side of Equation (3) reflects the fact that the banker forgoes the net proceeds from working capital loans if he or she defaults. See Appendix C for details.

probability σ a banker continues its business. With *i.i.d.* probability $1 - \sigma$ a banker exits, transfers retained earnings to the household and becomes a worker in period $t+1$. Thus, banker exit is a simple way to capture dividend payouts to the household.

At the *end* of period t, surviving banks have the option to raise new equity, e_t. In particular, after the bank finds out whether it has received the exit shock, in the case that it continues (with probability σ) it can pay cost $C(e_t, n_t)$ to raise new equity, e_t, from the household, which will be available in $t+1$ to fund the purchase of risky securities. In the case the bank exits at the end of t (with probability $1 - \sigma$), it does not have the option to issue new equity.[18] The equity issuance cost is meant to capture in a simple way the actual costs and frictions in the process of raising equity that banks face—for example, the costs of finding new investors or the frictions involved in the process of creating and selling new shares.[19]

Accordingly, the total net worth available for surviving banks in $t+1$ is given by

$$n_{t+1} = R_{K,t+1}Q_t s_t - R_t d_t + e_t \tag{4}$$

where $R_{K,t+1}$ is the gross rate of return on a unit of the bank's assets from t to $t+1$.

The Banker's Problem

The bank pays dividends only when it exits. If the exit shock realizes, the banker exits at the *beginning* of $t+1$, so it does not make any more working capital loans—it simply waits for the risky loans to mature and then pays the net proceeds to the household. If the bank continues, it has an option to raise equity. The objective of the bank is then to maximize expected terminal payouts to the household, net of the equity transferred by the household and of the cost of the transfer $C(e_t, n_t)$.

Formally, the bank chooses state-contingent sequences $\{s_t, s_{W,t}, d_t, e_t\}$ to solve

$$V_t(n_t) = \max \ (1-\sigma)\mathbb{E}_t \Lambda_{t,t+1} \left(R_{K,t+1}Q_t s_t - R_t d_t \right) + \sigma \left\{ \mathbb{E}_t \Lambda_{t,t+1} \left[V_{t+1}(n_{t+1}) - e_t \right] - C(e_t, n_t) \right\} \tag{5}$$

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[18]As long as the cost of raising equity is positive, for an exiting bank it would never pay to raise equity, as the new equity would simply be transferred back to the household.

[19]Alternatively, $C(e_t, n_t)$ can be interpreted as representing a cost of lowering net dividend payouts. In the model banks pay dividends with a fixed *ex ante* probability $1 - \sigma$, so in expectation—before the exit shock is realized—"net dividends" (dividend payouts net of new equity raised) equal $(1 - \sigma)\mathbb{E}_t(R_{K,t+1}Q_t s_t - R_t d_t) - \sigma e_t$. The cost $C(e_t, n_t)$ of increasing e_t is then akin to a cost of lowering net dividend payouts.

subject to

$$Q_t s_t + R_{t-1} d_{t-1} \leq R_{K,t} Q_{t-1} s_{t-1} + d_t + (R_{L,t} - R_{t-1}) s_{W,t} + e_{t-1} \tag{6}$$

and the incentive constraint given in equation (3), where $\Lambda_{t,t+1}$ is the household's stochastic discount factor, which is equal to the marginal rate of substitution between consumption at date $t+1$ and t. Equation (6) is the bank's budget constraint, stating that the bank's expenditures (consisting of asset purchases, $Q_t s_t$, and repayment of external financing, $R_{t-1} d_{t-1}$) cannot exceed its revenues (stemming from payments of previous-period loans, $R_{K,t} Q_{t-1} s_{t-1}$, new external financing, d_t, net proceeds from intraperiod working capital loans, $(R_{L,t} - R_{t-1}) s_{W,t}$, and new equity, e_{t-1}).

To solve the banker's problem, we first define the "working capital wedge", $\Delta_{L,t} \equiv R_{L,t} - R_{t-1}$), and the bank's balance sheet identity, $Q_t s_t \equiv n_t + d_t + \Delta_{L,t} s_{W,t}$. The latter equation states that risky loans are funded by the sum of net worth, external borrowing, and net proceeds from intraperiod loans.

Combining the bank's budget constraint, equation (6), with the bank balance sheet identity, we obtain the law of motion for net worth:

$$n_t = (R_{K,t} - R_{t-1}) Q_{t-1} s_{t-1} + R_{t-1} (\Delta_{L,t-1} s_{W,t-1} + n_{t-1}) + e_{t-1} \tag{7}$$

We then guess that the value function is linear in net worth, $V_t(n_t) = \alpha_t n_t$. Define

$$\mu_t \equiv \mathbb{E}_t[\Lambda_{t,t+1}(1 - \sigma + \sigma \alpha_{t+1})(R_{K,t+1} - R_t)] \tag{8}$$

$$\nu_t \equiv \mathbb{E}_t[\Lambda_{t,t+1}(1 - \sigma + \sigma \alpha_{t+1})] R_t \tag{9}$$

$$\nu_{e,t} \equiv \mathbb{E}_t[\Lambda_{t,t+1}(\alpha_{t+1} - 1)] \tag{10}$$

Note that α_{t+1}, capturing the value to the bank of an extra unit of net worth the following period, acts by "augmenting" banks' stochastic discount factor (SDF) so that their effective SDF is given by $\Lambda_{t,t+1}(1 - \sigma + \sigma \alpha_{t+1})$. The variable $\nu_{e,t}$ denotes the *net* value today of a transfer by the household that increases bank net worth tomorrow by one unit. In the decision to raise equity, the bank trades off the benefit $\nu_{e,t}$ against the issuing cost.

With these definitions, the problem simplifies to

$$\alpha_t n_t = \max_{s_t, s_{W,t}, e_t} \mu_t Q_t s_t + \nu_t \Delta_{L,t} s_{W,t} + \nu_t n_t + \sigma [\nu_{e,t} e_t - C(e_t, n_t)] \tag{11}$$

15

subject to the incentive constraint:

$$\mu_t Q_t s_t + \nu_t \Delta_{L,t} s_{W,t} + \nu_t n_t + \sigma \left[\nu_t e_t - C(e_t, n_t) \right] \geq \theta \left[Q_t s_t - \Delta_{L,t} s_{W,t} \right] + \omega s_{W,t} \quad (12)$$

Define $x_t \equiv \frac{e_t}{n_t}$, and assume that the equity cost takes the following form: $C(e_t, n_t) = \frac{\kappa}{2} x_t^2 n_t$. The first order conditions for s_t, $s_{W,t}$ and x_t, respectively, are as follows:[20]

$$(1 + \lambda_t)\mu_t = \lambda_t \theta \quad (13)$$

$$(1 + \lambda_t)\nu_t \Delta_{L,t} = \lambda_t(\omega - \theta \Delta_{L,t}) \quad (14)$$

$$\nu_{e,t} = \kappa x_t \quad (15)$$

where λ_t represents the Lagrange multiplier on the incentive constraint.

In our calibration below we set $\omega = \theta$, as a plausible benchmark case in which the recovery rates from working capital and from risky loans are the same. In that case, the following relationship obtains:

$$\frac{\Delta_{L,t}}{1 - \Delta_{L,t}} = \frac{\mu_t}{\nu_t} \quad (16)$$

Equation (16) states that the working capital wedge, $\Delta_{L,t}$, is a monotonic increasing function of the banks' excess return on risky loans, μ_t, divided by the value of net worth today, ν_t.[21] When the constraint does not bind (i.e., when $\lambda_t = 0$), we have $\mu_t = \Delta_{L,t} = 0$, and thus $\alpha_t = \nu_t + \frac{\sigma\kappa}{2}x_t^2$. When the constraint binds, $\mu_t > 0$, $\Delta_{L,t} > 0$ and $\alpha_t = \mu_t \phi_t + \nu_t + \frac{\sigma\kappa}{2}x_t^2$; bank asset funding is given by the constraint at equality, $Q_t s_t + (1 - \Delta_{L,t})s_{W,t} = \phi_t n_t$, where ϕ_t is the maximum leverage allowed for the bank.

Rearranging the incentive constraint, after imposing the optimality condition for working capital loans, equation (16), maximum leverage can be expressed as follows:

$$\phi_t = \frac{\nu_t + \frac{\sigma\kappa}{2}x_t^2}{\theta - \mu_t} \quad (17)$$

Since bankers' problem is linear, we can easily aggregate across banks. The law

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[20]The complete banker's problem is described in Appendix D.

[21]Note that to a first order, $\frac{\mu_t}{\nu_t} \approx \mathbb{E}_t \left[\Lambda_{t,t+1} \left(R_{K,t+1} - R_t \right) \right]$.

of motion for aggregate net worth is the following:

$$N_t = \sigma \left[(R_{K,t} - R_{t-1}) \underbrace{Q_{t-1}K_{t-1}}_{Q_{t-1}S_{t-1}} + x_{t-1}N_{t-1} + R_{t-1}\left(\Delta_{L,t-1}S_{W,t-1} + N_{t-1}\right) \right] + (1-\sigma)\xi Q_{t-1}K_{t-1}$$

(18)

3.2.1 The Choice of Equity Issuance

From the first order condition for equity issuance:

$$\underbrace{\mathbb{E}_t \left[\Lambda_{t,t+1} \left(\overbrace{\mu_{t+1}\phi_{t+1} + \nu_{t+1} + \frac{\sigma\kappa}{2}x_{t+1}^2}^{\alpha_{t+1}} - 1 \right) \right]}_{\nu_{e,t}} = c'(x_t)$$

(19)

The left hand side of equation (19) captures the marginal benefit for the bank of issuing one extra unit of equity, while the right hand side captures the marginal cost. Since the banker is ultimately a member of the household, the left-hand side captures the benefit of transferring a unit of resources from the household to the bank. Note that if the incentive constraint was never to bind in the future, the benefit of such transfer would be zero: we would have $\mu_{t+i} = 0$ for all $i \geq 1$, which from equations (9) and the Euler equation for riskless debt, $\mathbb{E}_t(\Lambda_{t,t+1}R_t) = 1$, implies a solution with $\nu_{t+i} + \frac{\sigma\kappa}{2}x_{t+i}^2 = 1$ for $i \geq 1$. Therefore, the value of equity issuance $\nu_{e,t}$ would be zero, and the bank would choose not to issue.

Conversely, if the constraint is expected to bind in the future (either in $t+1$ or in subsequent periods) we have $\nu_{e,t} > 0$. To the extent that there is a positive probability of the constraint binding in the future (as will be the case in our calibrated model), the value of issuing equity will always be positive for the bank. In that case, if there were no costs of equity issuance (i.e., if $c(x) = 0$ for all x) the net benefit of equity issuance would always be positive, providing incentives to issue without bound. The presence of cost thus helps ensure a finite and determinate rate of equity issuance.

3.3 Nonfinancial Firms

There are two categories of nonfinancial firms: final goods firms and capital producers. In turn, within final goods firms we also distinguish between "capital storage"

firms and final goods producers, in order to clarify the role of bank credit used to purchase capital goods.

3.3.1 Final Goods Firms

We assume that there are two types of final goods firms: capital storage firms and final goods producers.[22] The first type of firm purchases capital goods from capital good producers, stores them for one period, and then rents them to final goods firms. The latter type of firm combines physical capital rented from capital goods firms with labor to produce final output. Importantly, capital storage firms have to rely on banks to obtain funding to finance purchases of capital, as explained below. In addition, final goods producers need to rely on banks to finance working capital.

In period $t-1$, a representative capital storage firm purchases K_{t-1} units of physical capital at price Q_{t-1}. It finances these purchases by issuing S_{t-1} securities to banks which pay a state-contingent return $R_{K,t}$ in period t. At the beginning of period t, the realization of the capital quality shock ψ_t determines the effective amount of physical capital in possession of the firm, given by $e^{\psi_t} K_{t-1}$. The firm rents out this capital to final goods firms at price Z_t, and then sells the undepreciated capital $(1-\delta)e^{\psi_t} K_{t-1}$ in the market at price Q_t. The payoff to the firm per unit of physical capital purchased is thus $e^{\psi_t}[Z_t + (1-\delta)Q_t]$. Given frictionless contracting between firms and banks, it follows that the return on the securities issued by the firm is given by the following (Note that this equation implies that capital storage firms make zero profits state-by-state): $R_{K,t} = e^{\psi_t} \frac{Z_t + (1-\delta)Q_t}{Q_{t-1}}$.

The capital quality shock $\psi_t \sim N(0, \sigma_\psi)$, which provides a source of fluctuations in returns to banks' assets, is a simple way to introduce an exogenous source of variation in the value of capital.[23] These variations are enhanced by the movements in the endogenous asset price Q_t triggered by fluctuations in ψ_t.

In the aggregate, the law of motion for capital is given by

$$K_t = I_t + (1-\delta)e^{\psi_t} K_{t-1} \tag{20}$$

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[22]Ljungqvist and Sargent (2012) present a similar structure with two types of firms (see Chapter 12). Firms of type I and II in their notation correspond to our final goods producers and capital storage firms, respectively.

[23]This may be thought of as capturing some form of economic obsolescence. Gertler et al. (2012) provide an explicit microfoundation of fluctuations in capital quality ψ_t based on time-varying obsolescence of intermediate goods.

Final goods firms produce output Y_t using capital and labor: $Y_t = A_t F(e^{\psi_t} K_{t-1}, H_t)$, where A_t is a shock to the total factor productivity (TFP), which is assumed to follow an AR(1) process in logs: $\log(A_t) = \rho_A \log(A_{t-1}) + \epsilon_{A,t}$ where $\epsilon_{A,t} \sim N(0, \sigma_A)$.

Final good firms are subject to a working capital constraint: they borrow from banks in the beginning of period t to finance a fraction of the wage bill, $\Upsilon W_t H_t$, before production occurs. The bank charges gross interest rate $R_{L,t}$ per unit of working capital loans.[24]

Taking into account the working capital constraint, the first order conditions for labor and for physical capital are as follows:

$$A_t F_1(K_t, H_t) = Z_t \tag{21}$$

$$A_t F_2(K_t, H_t) = W_t \left[1 + \Upsilon(R_{L,t} - 1) \right] \tag{22}$$

The credit spread is then given by the expected return on nonfinancial firm securities, $\mathbb{E}_t(R_{K,t+1})$, net of the riskless rate, R_t. In our simulations below, we report an annualized credit spread, as in the data. We annualize the spread in any period t by cumulating the quarterly spreads over the four subsequent periods.[25]

3.3.2 Capital Producers

Capital producers make new capital using final output and are subject to adjustment costs. They sell new capital to firms at the price Q_t. Given that households own capital producers, the objective of the capital producer is to choose $\{I_t\}$ to maximize the expected discounted value of profits:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{t+i} \left\{ Q_t I_t - \left[1 + f\left(\frac{I_t}{I_{t-1}}\right) \right] I_t \right\} \tag{23}$$

The price of capital goods is equal to the marginal cost of investment goods:

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[24]In equilibrium, the total amount of working capital loans by banks is given by $S_{W,t} = \Upsilon W_t H_t$.

[25]One consideration to note when comparing the model credit spread to its empirical counterpart is that the latter likely reflects in part expected default costs on corporate bonds, while in the model there is no explicit default by nonfinancial firms. It would be straightforward to extend the model to allow for corporate default—as in, for example, Bernanke et al. (1999). However, given the finding by Gilchrist and Zakrajsek (2012) that the excess bond premium accounts for a sizable part of the variation of credit spreads, and with the goal of preserving model simplicity, we choose to abstract from default by nonfinancial firms.

$$Q_t = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}}f'\left(\frac{I_t}{I_{t-1}}\right) - E_t\Lambda_{t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 f'\left(\frac{I_{t+1}}{I_t}\right) \qquad (24)$$

3.4 International Capital Markets

We follow Schmitt-Grohe and Uribe (2003) and assume that small open economy is subject to debt-elastic interest rate premium in the international markets.

$$R_t = \frac{1}{\beta} + \varphi(e^{\frac{B_t^*}{Y} - \bar{b}} - 1) + e^{R_t^* - 1} - 1 \qquad (25)$$

where $\bar{b}$ governs the steady state foreign debt to GDP ratio and R_t^* is a shock to the country interest rate, which is assumed to follow an AR(1) process in logs: $log(R_t^*) = \rho_R \log(R_{t-1}^*) + \epsilon_{R,t}$, where $\epsilon_{R,t} \sim N(0, \sigma_R)$.

3.5 Resource Constraint and Market Clearing

The resource constraint and the balance of payments equations, respectively, are given by:

$$Y_t = C_t + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right)\right]I_t + \sigma\frac{\kappa}{2}x_t^2 N_t + NX_t \qquad (26)$$

$$R_{t-1}B_{t-1}^* - B_t^* + (R_{t-1} - 1)\Upsilon W_t H_t = NX_t \qquad (27)$$

where NX stands for net exports. (Proved in Appendix E.)

4 Functional Forms and Calibration

In this section we describe, in turn, the functional forms and the parameter values used in the model simulations.

4.1 Functional Forms

The functional forms of preferences, production function, and investment adjustment cost are the following:

$$U(C_t, H_t) = \frac{\left(C_t - \chi \frac{H_t^{1+\epsilon}}{1+\epsilon}\right)^{1-\gamma} - 1}{1 - \gamma} \tag{28}$$

$$F(K_t, H_t) = (e^{\psi_t} K_{t-1})^{\eta} H_t^{1-\eta} \tag{29}$$

$$f\left(\frac{I_t}{I_{t-1}}\right) = \frac{\vartheta}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 \tag{30}$$

The utility function, equation (28), is defined as in Greenwood et al. (1988), which implies non-separability between consumption and leisure. This assumption eliminates the wealth effect on labor supply by making the marginal rate of substitution between consumption and labor independent of consumption. The parameter γ is the coefficient of relative risk aversion, and ϵ determines the wage elasticity of labor supply, given by $1/\epsilon$. The production function, equation (29), takes the Cobb-Douglas form. The coefficient η is the elasticity of output with respect to capital. Equation (30) is the flow investment adjustment cost function, with the investment adjustment cost parameter given by ϑ.

4.2 Calibration

Table 2 reports the parameter values. The model includes ten conventional preference and technology parameters, for which we choose values that are relatively standard in the literature. We set the discount factor, β, to 0.985, implying an annual real country interest rate of 6%.[26] We set the risk aversion parameter, γ, to 2, a standard value in the literature (for example, Mendoza (1991), Uribe and Yue (2006), and Mendoza (2010)). The Frisch labor supply elasticity, given by $1/\epsilon$, is set to 8, a value that is above the range typically found in the literature. As in Gertler and Kiyotaki (2010), this relatively high value represents an attempt to compensate for the absence of frictions such as nominal wage and price rigidities, which are typically included in quantitative DSGE models. While our framework excludes these frictions

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[26]Given that the risk-free real interest rate is around 4% annually, the countries in our sample are assumed to pay 2% premium per annum, on average, to borrow in the international financial markets, which is roughly in line with the data.

Table 2: Calibration

Parameter	Symbol	Value	Source/Target
Conventional			
Discount factor	β	0.985	Interest rate (6%, ann.)
Risk aversion	γ	2	Standard RBC value
Inverse Frisch elast.	ϵ	1/8	Frisch lab. sup. elast. (inv)
Labor disutility	χ	2.07	Steady state labor (30%)
Capital share	η	0.33	Standard RBC value
Capital depreciation	δ	0.025	Standard RBC value
Investment adj. cost	ϑ	1	Gertler and Kiyotaki (2010)
Debt elast. of interest rate	φ	0.01	Standard SOE-RBC value
Reference debt/output ratio	$\bar{b}$	1	Steady state B/Y of 100%
Frac. Wage bill paid in advance	Υ	1	Neumeyer and Perri (2005)
Financial Intermediaries			
Survival rate	σ	0.95	Exp. survival of 5 yrs
Transfer rate	ξ	0.0001	Small value
Fraction divertable	θ	0.24	{ Frequency of crises around 5%,
Cost of raising equity	κ	28	Leverage of 6.5 }
Shock Processes			
Persistence of interest rate	ρ_R	0.8694	
SD of interest rate innov. (%)	σ_R	0.14	
Persistence of TFP	ρ_A	0.92	
SD of TFP innov. (%)	σ_A	0.375	
SD of capital quality (%)	σ_ψ	0.75	

to preserve simplicity, they likely have a role in accounting for employment and output volatility in the countries we study. Accordingly, as Gertler and Kiyotaki (2010) do, we partly compensate for their absence by setting a relatively high elasticity of labor supply.

Turning to technology parameters, we set the capital share, η, the depreciation of capital, δ, and the parameter governing investment adjustment costs, ϑ, to 0.33, 0.025, and 1, respectively. These are standard values in the literature. We follow Schmitt-Grohe and Uribe (2003) and assign a small value to the parameter φ, measuring the sensitivity of the country interest rate to deviations of the foreign debt to GDP ratio from its long-run value. The reference debt to output ratio in the country interest rate function, equation (25) is set to 1, which yields a ratio of external debt to GDP of 25%

annually—a conservative estimate. Finally, we set the working capital requirement, Υ, to 1, similar to Neumeyer and Perri (2005) and Uribe and Yue (2006).

We then assign values to the four parameters relating to financial intermediaries: the survival rate of bankers, σ, the transfer to entering bankers, ξ, the fraction of assets that bankers can divert, θ, and the parameter determining the cost of raising equity, κ. We calibrate σ to 0.95 as in Gertler and Kiyotaki (2015), implying that bankers survive for about 5 years on average. We set the transfer rate ξ to a very small number, to ensure that this parameter does not alter the results while still allowing the entering bankers to start operations.[27] We then calibrate the remaining two parameters, θ and κ, to hit two targets: a leverage ratio of about 6.5 in the stochastic steady state, and a frequency of financial crises of about 4% annually. The target leverage ratio is a crude estimate of the average financial sector leverage for the countries in our sample.[28] The target for crisis frequency is taken from Schularick and Taylor (2012).[29] These targets yield a value for θ of 0.24, and for κ of 28.

Turning to the exogenous shocks, we estimate a first order autoregressive process (with fixed effects) for the real country interest rate using panel data from the six countries in our sample, as in Akinci (2013) and Uribe and Yue (2006). We use the methodology proposed in Schmitt-Grohe and Uribe (2016) to calculate real country interest rates, R_t, in each country in our sample. Specifically, we construct time series for the quarterly real gross country interest rate using the following equation: $R_t = (1+i_t)E_t\left(\frac{1}{\pi_{t+1}}\right)$, where i_t denotes the nominal sovereign borrowing rate faced by the countries in our sample (in units of their own currency) in international financial markets, and π_{t+1} denotes CPI inflation rate in each country. We measure $E_t\left(\frac{1}{\pi_{t+1}}\right)$ as the fitted component of a regression of $\frac{1}{\pi_{t+1}}$ onto a constant and two lags of inflation.[30]

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[27]We verified that setting ξ to smaller values has virtually no effect on the results reported.

[28]We obtain this estimate using balance sheet data for financial institutions from Haver Analytics (who in turn retrieve it from national central banks). Because the liability side of banks in our model is quite stylized—including just short-term noncontingent deposits and net worth— and does not fully capture the complexity of banks' liabilities in practice, we approximate aggregate financial sector equity in two different ways, that we interpret as a lower and an upper bound. First, we simply set net worth equal to the category "capital" in the balance sheet data. Second, we compute equity as the sum of capital and the category "other liabilities." The latter includes items with some hedging properties, for example provisions and subordinated debt. We then compute leverage as the ratio of total domestic nonfinancial assets and each of the two measures of capital. We obtain a sample average across the six countries of 8.4 using the first definition of net worth, and of 4.5 using the second. Our target of 6.5 then represents the mid-point of these two estimates.

[29]Schularick and Taylor (2012) report an annual probability of crises of four percent since the 1970s.

[30]The results are robust to using higher lags of inflation in calculating real interest rates. Data

Similarly, we estimate a first order autoregressive process for TFP using a quarterly panel for the countries in our sample.[31] Finally, we calibrate the volatility of the capital quality shock (which is assumed to be *iid*) so that the model delivers a standard deviation of financial sector equity (relative to trend) of about 14%, roughly the average value in our sample.

5   Model Analysis

This section illustrates the features of the model via a series of numerical experiments. To solve the model, we use a version of the parameterized expectations algorithm, described in Appendix G.[32] The approach consists in using parametric functions to approximate the model's one-step-ahead conditional expectations as functions of the aggregate state vector. As emphasized by Christiano and Fisher (2000), in models with occasionally binding constraints this approach can be superior to approximating the model's policy functions, since the latter exhibit pronounced kinks that are hard to approximate, while the conditional expectations are likely to be smoother.[33]

Because our method allows us to solve the model globally, we can capture the nonlinear "financial accelerator" dynamics arising as a result of the occasionally binding incentive constraint. Appendix I illustrates this point with the help of the model's policy functions. In addition, we can appropriately capture the risk-taking behavior of banks, as the method fully accounts for shock uncertainty.

This section is organized as follows: we first report a set of long-run moments from long simulations of the model, and compare them against the data. We then analyze the model-generated series for credit spreads, and show that their properties resemble those in the data that we highlighted in section 2. We then identify "financial crisis" episodes in the model, and also compare their characteristics to the data.

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for sovereign bond yields are obtained from Bloomberg.

[31]TFP data is available at an annual frequency, obtained from the Total Economy Database. We interpolate it to a quarterly frequency using the method by Boot et al. (1967).

[32]See Marcet and Lorenzoni (1998) and Christiano and Fisher (2000) for descriptions of the method of parameterized expectations applied to solving rational expectations models. In Appendix H we assess the accuracy of our method by computing Euler equation residuals.

[33]In Appendix I, we show that this is indeed the case in our setting.

5.1 Long-Run Moments

We begin by reporting some standard business-cycle moments from our calibrated model, shown in Table 3. The model does an overall good job at reproducing the key business cycle facts in the data. It generates a realistic amount of output volatility: the standard deviation of output (in log deviations from HP trend) is about 1.7 percent, compared to roughly 1.4 percent in the data. The model can also capture the fact that consumption is nearly as volatile as output in the data (the standard deviation of consumption relative to that of output is 0.97 in the model versus 0.92 in the data), and that investment is substantially more volatile than output. Here the model overstates the investment volatility: its standard deviation is about 5 times larger than that of output in the model, compared to about 3 times larger in the data. By construction, the volatility of bank equity in the model also resembles the data (recall that we calibrated the capital quality shock to deliver a realistic volatility of bank equity). On the other hand, the standard deviation of the credit spread, not targeted, is also close to the data.

The model also does a good job at delivering the correct cyclicality of all variables. It generates strongly procyclical investment, consumption, employment and bank equity—their correlations with output are well aligned with the data. Our data features a somewhat countercyclical net exports to GDP ratio (correlation of about -0.3) which the model delivers as well. Finally, the model's credit spread is also countercyclical, as in the data.

Table 4 presents the stochastic steady state of the model, along with some statistics relating to financial crises from our baseline simulation. The stochastic steady state is defined as the point at which the economy settles in the absence of exogenous shocks, but in which agents still expect that shocks might occur in the future.[34] As shown in the upper panel of the table, banks' leverage ratio in the stochastic steady state is lower than the maximum leverage allowed by the incentive constraint, reflecting that the constraint does not bind. As a result, the credit spread is relatively low, taking a value of only about 70 basis points annually. As we show below, when the constraint binds the spread will jump to much higher levels.

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[34]To calculate the stochastic steady state, we simulate the economy for a large enough number of periods without any exogenous shocks, until the system converges to a point at which all endogenous variables are constant. This approach has also been used by Gertler et al. (2012) and Aoki et al. (2015).

Table 3: Business Cycle Statistics, Model versus Data

Variable		SD (%)	SD rel. to Output	Corr. with Output
Output	*Model*	1.696	1.000	1.000
	Data	1.362	1.000	1.000
Consumption	*Model*	1.652	0.974	0.979
	Data	1.232	0.924	0.863
Investment	*Model*	8.981	5.294	0.600
	Data	3.778	2.819	0.899
Employment	*Model*	1.682	0.992	0.983
	Data	1.282	0.937	0.570
NX/GDP	*Model*	1.562	0.921	-0.317
	Data	0.585	0.448	-0.334
Bank Equity	*Model*	13.911	8.200	0.639
	Data	15.457	11.658	0.566
Credit Spread	*Model*	1.102	0.649	-0.487
	Data	0.803	0.586	-0.218

Note: Business cycle statistics for data calculated as the simple average across all the countries in our sample (Italy, Spain, Germany, France, UK and the US). All variables except the credit spread are detrended using the HP filter. The data are expressed in units of the GDP deflator.

Although the constraint is not binding at the stochastic steady state, banks anticipate that they may be constrained in the future, which then leads them to issue equity at a positive rate (of about 2 percent of their net worth per quarter). Given the calibrated bank exit rate of 5 percent, the value for banks' choice of equity issuance then implies a rate of "net dividend" payout to the household of about 3 percent. The rate of net dividend payout then helps determine the steady state leverage ratio, which takes a value of six and a half—one of the targets in our calibration.

How important is equity issuance in determining the banking sector's net worth? Rearranging the law of motion of aggregate net worth—equation (18)—in the stochas-

Table 4: Stochastic Steady State & Simulation Statistics

Variable	Symbol	Baseline Model	Model w/o Constraint
Output	Y	0.930	1.209
Consumption	C	0.720	0.931
Labor	L	0.339	0.429
Capital	K	7.192	9.929
Net worth	N	1.202	–
Equity issuance rate	x_t	0.023	–
Leverage ratio	$\frac{QS+(1-\Delta_L)S_W}{N}$	6.504	–
Maximum leverage ratio	ϕ	6.639	–
Spread (annualized, %)	$E(R_K) - R$	0.670	0.073
Foreign debt	B^*	0.901	1.187

Statistics

Prob. of binding const., 2-qtr-ahead (%)		16.04	–
Prob. of binding const., 1-yr-ahead (%)		35.69	–
Time at the constraint (% of quarters)		13.76	–
Binding const. event, mean duration (qtrs)		2.48	–
Crisis event, mean duration (quarters)		5.82	–
Number of crisis events per 100 years		4.40	–

tic steady state, and letting variables without a subscript refer to steady state:[35]

$$N = \underbrace{\frac{\sigma}{1-\sigma R}\left(R_K - R\right)QS}_{\text{retained earnings}} + \underbrace{\frac{\sigma}{1-\sigma R}xN}_{\text{equity issuance}} + \underbrace{\frac{1-\sigma}{1-\sigma R}\xi QS}_{\text{startup transfer}}$$

Here, the first term on the right-hand side is (long-run) retained earnings (arising from the positive, but small, steady-state spread $R_K - R$), the second term is equity issuance, and the third term is the start-up transfer. Given our calibration, roughly two fifths of long-run net worth N are accounted for by retained earnings, three fifths by equity issuance, and a negligible fraction by start-up transfers (recall that we set the start-up transfer rate, ξ, to a very small number). Thus, in our framework endogenous equity issuance plays a quantitatively dominant role in determining long-

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[35]Here we have set $\Delta_L = 0$, by virtue of (16) together with $\mu = 0$, indicating that in steady state banks do not make excess returns on working capital loans.

run bank net worth.

It is important to emphasize that absent the prospect of a binding constraint in the future, banks would prefer to set $x = 0$. Instead, in our model with agency frictions, banks expect that they will be constrained in some future states, in which therefore an extra unit of net worth will be highly valuable. This then leads them to issue a positive amount of equity today. Thus, what ultimately drives banks' equity issuance is a precautionary motive, leading them to be better capitalized (i.e. have higher net worth) than they would if their rate of equity issuance was zero.

What is the impact of the agency friction on the economy's long-run equilibrium? The last column in Table 4 reports the stochastic steady state of an economy in which banks cannot default—implying there is no incentive constraint in their optimization problem—and which is otherwise calibrated exactly as our baseline model. Absent agency frictions, the model reduces to a frictionless small open economy model without distortions stemming from the financial sector.[36] Unlike in our baseline model with agency frictions, financial sector net worth becomes irrelevant for real outcomes, and so banks are simply "veils" that channel funds from households and foreigners to domestic nonfinancial firms. As seen in the Table, the credit spread in the stochastic steady state of this economy is still positive at about 7 basis points, but its value is only a fraction of that in our baseline model with the agency frictions (despite the fact that in the steady state of the latter, the constraint is not binding).[37] A direct result of the lower steady-state spread is a larger amount of physical capital—about 38 larger in the unconstrained model than in our baseline model—and, consequently, higher output and consumption.

Turning back to our baseline model with agency frictions, the lower panel of Table 4 shows that although banks are unconstrained in the stochastic steady state, the probability that the constraint might bind in the future is substantial: the probability of the constraint binding for at least one period within the next two quarters is about

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[36] To see this, note that absent banks' incentive constraint we must have perfect arbitrage at all times: $\mu_t = \Delta_{L,t} = 0 \ \forall t$, which then implies $\alpha_t = \nu_t = 1 \ \forall t$ (using the household's Euler equation), and $x_t = 0 \ \forall t$. The Euler equation for capital is then the standard $\mathbb{E}_t (\Lambda_{t+1} R_{K,t+1}) = 1$.

[37] When the constraint does not bind, as in the steady state, the one-quarter-ahead credit spread is given by $\mathbb{E}_t (R_{K,t+1} - R_t) = \frac{-Cov_t(\mathcal{M}_{t+1}, R_{K,+1})}{\mathbb{E}_t(\mathcal{M}_{t+1})}$, where $\mathcal{M}_{t+1}$ denotes the relevant stochastic discount factor: $\mathcal{M}_{t+1} = \Lambda_{t+1}(1 - \sigma + \sigma\alpha_{t+1})$ in our baseline model, and $\mathcal{M}_{t+1} = \Lambda_{t+1}$ in the model without agency frictions. The "augmented" stochastic discount factor $\Lambda_{t+1}(1 - \sigma + \sigma\alpha_{t+1})$ is much more countercyclical than Λ_{t+1}, because the marginal value of bank net worth α_{t+1} is strongly countercyclical. This then leads to a much larger credit spread in steady state.

16 percent. The same probability over the next year is much higher, 35 percent. The overall fraction of time the economy spends at the constraint in the simulation is relatively low, just 13 percent of quarters, and the average spell of a binding constraint lasts for about 2.5 quarters.

The bottom two rows of Table 4 present statistics related to financial crises in our model. We identify a financial crisis event as a situation in which the leverage constraint binds for at least four consecutive quarters, following the empirical evidence in Laeven and Valencia (2012) showing that actual banking crisis episodes tend to last for at least one year.[38] In our model, accordingly, financial crisis events last for 5.8 quarters on average, and the number of crisis events per 100 years is 4.4—in line with the empirical financial crisis probability that we used as one of the targets in our calibration of financial sector parameters.

5.2   Credit Spreads and Financial Crises

In this section, we evaluate our model's quantitative performance in matching the three facts we presented in section 2. More precisely, we show that our baseline model with the occasionally binding leverage constraint can account for the facts that credit spreads display occasional large spikes, and that spreads are more strongly counter-cyclical when they are relatively elevated. We also show how our model produces occasional financial crisis episodes that are associated with elevated levels of credit spreads, sharp losses in banking sector equity, and with deep recessions, in a manner that is quantitatively consistent with the evidence.
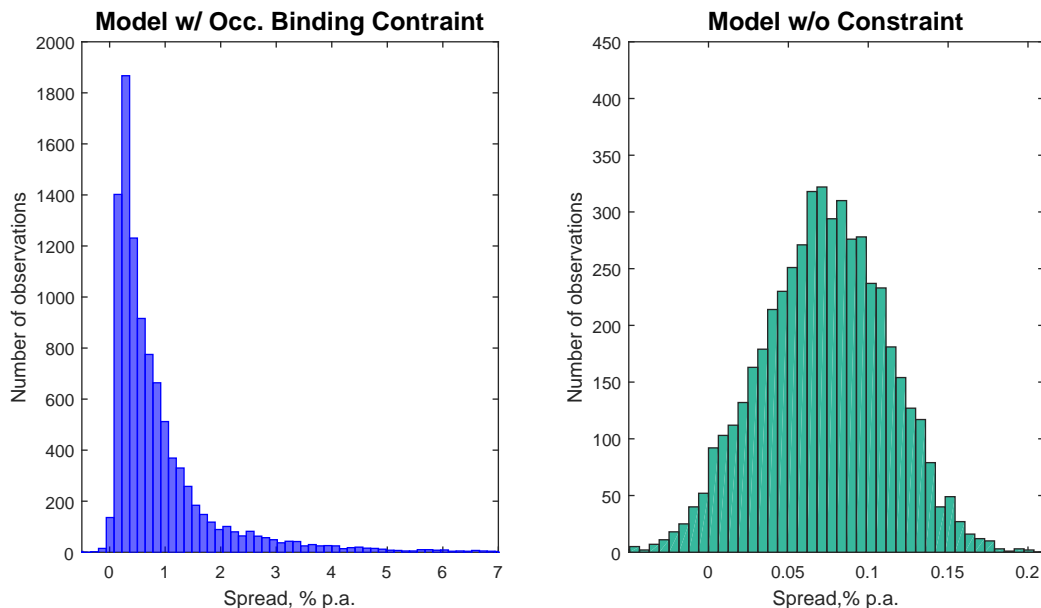
Figure 4 shows a histogram of credit spreads in our baseline model (the left panel), and compares it with the credit spreads implied by the model without the constraint (the right panel). It is immediately apparent from the comparison of the two panels that our baseline model delivers an asymmetric distribution of credit spreads, as in the data, while the frictionless model does not. The first two rows of Table 5 confirm this result: the baseline model delivers excess skewness and kurtosis, while in the frictionless model those moments are close to those of a normal distribution. Thus, our baseline model can explain the occurrence of the occasional and large increases in credit spreads that we observe in the data (see Figure 1 in section 2). In addition,

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[38]Based on data on financial crises in Laeven and Valencia (2012), the duration of the identified crisis events ranges from one year (for example, the 1988 financial crisis in the U.S. lasted a year) to around five years or more (for example, the financial crisis in Spain in 1977 lasted five years).

the magnitudes of the credit spread in our baseline model resemble those in the data, while in the unconstrained model they are much smaller.

Figure 4: Histogram of Credit Spreads

Note: The figure shows the histograms of credit spreads from a 10,000-period model simulation. The left panel corresponds to our baseline model with the occasionally binding constraint, and the right panel corresponds to a frictionless model without the constraint.

Table 5: Credit Spreads, Moments

	Baseline Model	Model w/o Constraint
Skewness	3.66	-0.05
Kurtosis	23.18	2.84
ρ^-	-0.27	0.11
ρ^+	-0.38	0.13

Note: Credit spreads skewness, kurtosis, and correlation with GDP conditional on negative (ρ^-) and positive (ρ^-) spread deviations from mean, for baseline model (left column) and model without agency frictions (right column).

As seen in Figure 5 and in the bottom two rows of Table 5, our model also captures the asymmetric relation between credit spreads and economic activity: when spreads are relatively elevated, they tend to be more strongly associated with real activity

Figure 5: Credit Spreads and GDP
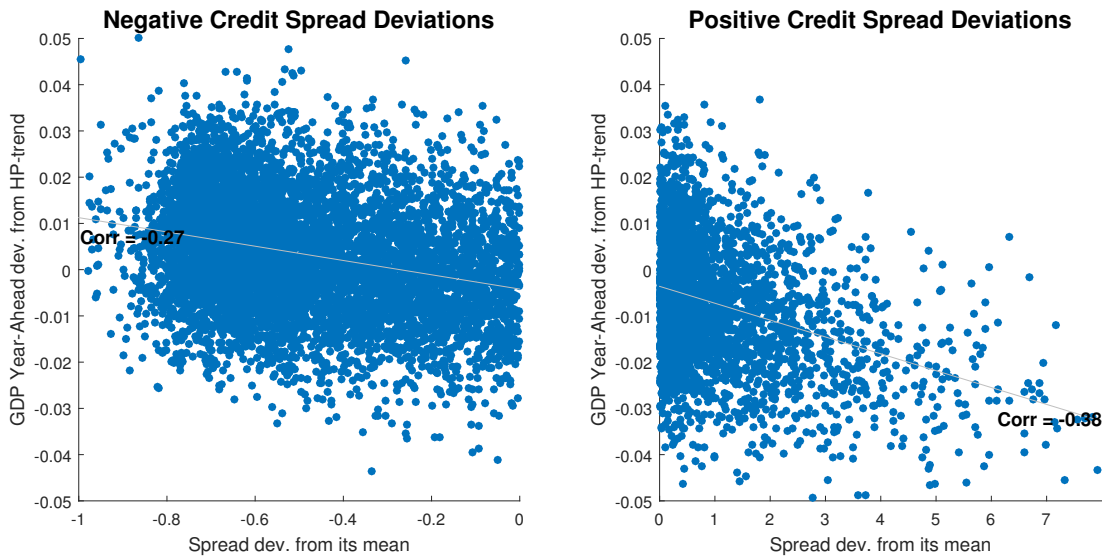
Note: The panels show the relation between GDP and credit spreads in a 10,000-period model simulation. As in Figure 2, the vertical axis shows year-ahead average deviations of GDP from its HP trend, and the horizontal axis shows deviations of the spread from its mean. The left panel contains negative spread deviations, and the right panel contains positive deviations.

than when they are relatively compressed. In particular, considering only the subsample of quarters in our simulation in which the spread is above the mean yields a correlation between deviations in spreads and in real economic activity (calculated as year-ahead deviation of real GDP from its HP trend, as in the data) of about -0.38, compared with -0.43 in the data (recall Figure 2). By contrast, in the complementary subsample in which credit spreads are relatively subdued (i.e. below the mean) the correlation is substantially weaker, at only -0.27 (-0.13 in the data). From Table 5, note also that the frictionless model cannot reproduce this feature of the data: spreads are actually mildly procyclical, and their degree of cyclicality does not change when we compare negative and positive spread deviations.

As before, key to explaining the model's ability to generate this asymmetry is the occasionally binding incentive constraint: a binding constraint tends to be associated with elevated levels of the credit spread, and at the same time leads to amplified movements in real activity (via the financial accelerator). Conversely, times of low spreads tend to be times when the constraint is slack, in which therefore these amplification effects are absent. In this way the model delivers the asymmetric relation between spreads and activity identified in the data.

We next describe what the average financial crisis event looks like in our framework. As mentioned earlier, we define a crisis event as any instance in which banks' incentive constraint binds for at least four consecutive quarters—consistent with the empirical evidence in Laeven and Valencia (2012). We then consider a window that begins 8 quarters before the crisis and ends 8 quarters after. For each variable, we compute the average period-by-period across all the crisis events that we identify. We index the quarter with the highest level of financial stress in the episode (measured as the quarter in which the credit spread is at its peak) to $t = 0$, as we do in the data. We also normalize all series except credit spreads to their respective pre-crisis peaks.

Figure 6 displays the dynamics around the typical financial crisis episode. In the quarters leading up to the crisis, bank equity (first panel, right column) deteriorates sharply, by 60 percent relative to HP trend in about one and a half years. These equity losses eventually put banks up against their borrowing constraints, leading credit spreads (first panel, left column) to jump sharply: the spread increases from just below 2 percent annually to almost 6 percent in only two quarters. Along the way, with a binding constraint, the financial accelerator mechanism operates, with the drops in net worth, investment, and asset prices reinforcing each other. All told, investment at the trough is about 20 percent below trend in the simulation. Output drops by more than 5 percent around time 0 and remains depressed thereafter. Overall, our model generates financial crises consistent with the evidence: crisis periods feature severe disruption in financial intermediation, exemplified by large increases in credit spreads and sharp losses in bank equity, as well as plunges in domestic investment, consumption and output, with magnitudes that are consistent with the data.[39]

From the bottom panel in Figure 6, the crisis is ultimately the result of adverse realizations of all three exogenous shocks: crisis events are triggered by a negative sequence of capital quality shocks, together with an increase in country interest rates and a decrease in TFP. Note, however, that the realizations of the shocks that trigger the crisis are not abnormally large: at time 0, capital quality is down by about 0.9 percent (a little more than one standard deviation), the (log of) R^* is up by about

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[39]In the model, the trough in output occurs in the same period as the peak in the credit spread, while in the data output bottoms out several quarters after the spread peaks. This discrepancy is partly driven by the behavior of employment, which in the model reacts to credit spreads contemporaneously (via higher interest on working capital) but in the data it typically declines gradually after spreads peak. One way to address this would be to include labor adjustment costs, as a reduced-form way to capture labor market frictions such as search and matching and wage rigidity.

Figure 6: Average Financial Crisis: Model versus Data
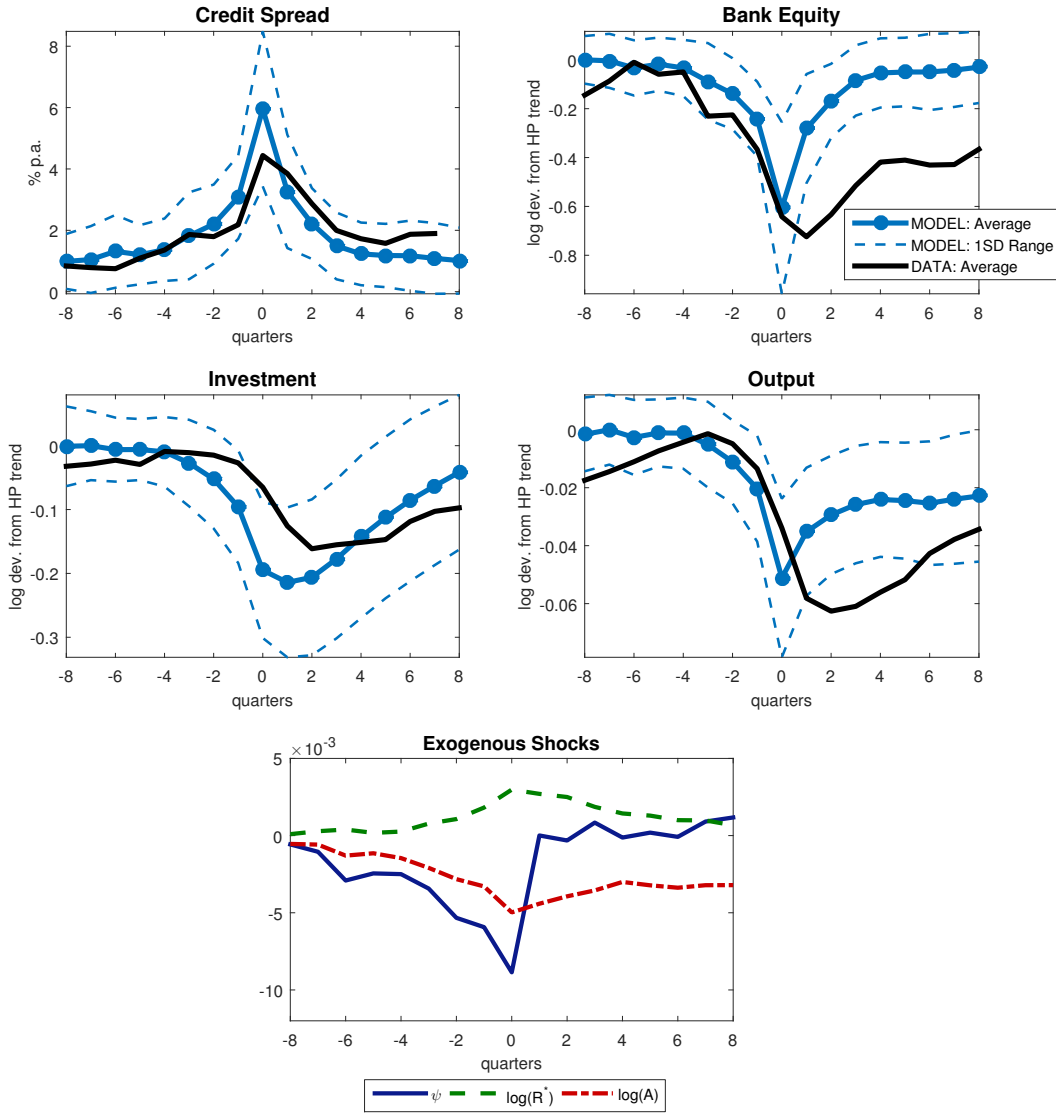
Note: We simulate the economy for 10,000 periods and compute averages across financial crisis events. A financial crisis is defined as an event in which banks' constraint binds for at least four consecutive quarters. The first four panels show the model counterparts (in blue circled lines) of the data reported in Figure 3, along with the data averages (in black). The last panel shows the average values of the exogenous variables in a crisis event.

0.3 percent (slightly more than one unconditional standard deviation) and the (log of) A is down by about 0.5 percent (less than one unconditional standard deviation). Thus, crises in the model are triggered not by unusually large adverse realizations of the shocks, but by a moderately adverse sequence of all the three shocks which push

the economy toward the constrained region and eventually trigger the constraint.[40]

6   Macroprudential Policy

This section explores the effectiveness of macroprudential polices within our framework. The goal is to analyze whether there is a role for the government to tilt banks' incentives in favor of having more net worth. The motivation for such policies is that banks may undervalue their equity capital when the financial sector is up against the incentive compatibility constraint.[41] When the constraint binds, a better-capitalized banking sector would imply smaller drops in asset prices and, in turn, smaller declines in aggregate net worth and more moderate increases in credit spreads. Since atomistic banks do not internalize these aggregate effects, their privately optimal choice of equity capital may be inefficiently low. Several authors have analyzed the role of this type of pecuniary externality in motivating some form of government intervention.[42] While much of this literature has considered policies aimed at reducing economy-wide borrowing (for example, via taxes on foreign debt), within our framework the key determinant of whether the economy enters into crisis mode is the net worth of the domestic financial sector.

Accordingly, we consider two types of policy directed at increasing banks' net worth. The first type of policy, which we call "government subsidy," features a regulatory subsidy to banks' equity issuance, financed by lump-sum taxes. The advantage of studying this policy is that it clearly illustrates the extent to which banks' privately optimal choice of equity issuance is socially inefficient. As we show, the degree of inefficiency is high, and the welfare gains from an appropriately chosen subsidy are large.

There are several reasons, however, why the above policy might be hard to implement in practice. For example, lump-sum taxes may not be available. It may also

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[40]Of course, the probability of obtaining an adverse sequence of realizations such as the line corresponding to ψ in Figure 6 is lower than the probability of a single adverse realization of one standard deviation.

[41]Unless otherwise stated, by "constraint" we refer to the incentive compatibility constraint, as presented in equation (3).

[42]For theoretical treatments see, for example, Lorenzoni (2008), Stein (2012), Jeanne and Korinek (2016), and Dávila and Korinek (2016). For quantitative analyses, see Jeanne and Korinek (2010), Bianchi (2011), Benigno et al. (2013), Bianchi and Mendoza (forthcoming) or Schmitt-Grohe and Uribe (2016).

be infeasible, due to political economy considerations, to subsidize banks by taxing households. This leads us to consider a second type of policy, consisting of a regulatory capital requirement that banks have to satisfy in normal times. In particular, we assume that when the incentive compatibility constraint does not bind, banks have to meet a limit on their leverage ratio set by the regulatory agency.[43] Unlike the government subsidy, this policy has the advantage of closely reproducing real-world policies such as the capital requirements implemented within the Basel framework.

6.1   Government Subsidy

We consider a government subsidy to banks equal to τ^s per unit of equity issued. The subsidy thus works to reduce the net cost of issuing equity for banks. Given the subsidy, the first-order condition for equity issuance becomes $\nu_{e,t} + \tau^s = c'(x_t)$. Thus, the policy induces banks to choose a higher rate of equity issuance x_t.

We restrict attention to the case where the subsidy is set to a constant. Of course, by allowing the subsidy to be time-varying, the welfare gains from an appropriately chosen subsidy scheme would be necessarily larger—the case we study thus provides a lower bound on the potential gains from this policy. On the other hand, allowing the government to finance the subsidy via nondistortionary taxation permits the welfare gains to be highest (within the class of constant subsidies). We focus on this case because we want to explore the scope for policy in affecting banks' privately optimal equity issuance decision, without imposing distortions on other margins.

The first two panels in Figure 7 show the effects of policy on the steady-state probabilities of the constraint binding (left panel) and on the frequency that the economy spends at the constraint (middle panel) for values of the subsidy τ^s ranging from zero to sixty percent. As the subsidy increases, both the probabilities of the constraint binding and the fraction of time spent at the constraint diminish monotonically. The reason is simple: the subsidy works to increase financial-sector net worth, thus making the latter less likely to reach the point at which banks' constraints start binding.

Next, we calculate the welfare gains from policy. Our metric is the unconditional expectation of welfare. For each value of the policy τ^s, we compute consumption-

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[43]Christiano and Ikeda (2014) also study the welfare implications of leverage restrictions on banks (see also Christiano and Ikeda (2016)). Their focus is on how these restrictions can improve on the private economy at the determistic steady state. By contrast, our framework emphasizes the dynamic benefits of leverage restrictions in terms of avoiding events of extreme financial distress.

Figure 7: Effects of Government Subsidy on Crisis Probabilities and Welfare
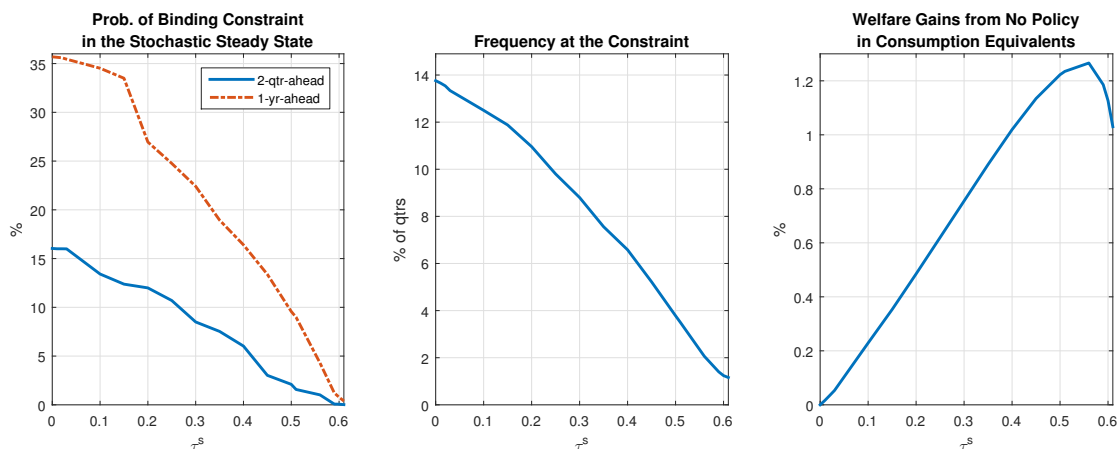
Note: The panels show, for a range of values of the government subsidy τ^s, the 2-quarter-ahead and 1-year-ahead probabilities of the constraint binding at least once (left panel), the percentage of quarters spent at the constraint in the simulation (middle panel) and the welfare gains from macroprudential policy in consumption equivalents (right panel).

equivalent welfare gains by finding the permanent increase in quarterly consumption in the no-policy economy required to make the household as well off as in the economy with policy.[44]

The policy raises welfare, for all values of the subsidy that we analyzed. Welfare gains peak when the subsidy is set to 56 percent. The welfare gains are large: at the peak, they reach 1.27 percent of quarterly consumption, a value that is above the range typically found in the literature.[45]

Table 6 reports the stochastic steady state, along with some simulation statistics, for the case in which the policy is set to the welfare-maximizing level ($\tau^s = 0.56$). For comparison, the Table also includes the statistics corresponding to the no-policy economy. The policy increases the steady-state equity issuance rate, x, by more than fifty percent, from 2.2 to 3.4 percent. This raises bank net worth and reduces leverage. As a consequence, the steady-state spread is reduced dramatically, from nearly 70 basis points to just 10 (recall that the spread is heavily affected by the

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[44]That is, for each value of τ^s, we find $\Delta(\tau^s)$ such that $\mathbb{E}\left\{\sum_{t=0}^{\infty} \beta^t U\left(C_t^{NP}\left[1 + \Delta(\tau^s)\right], H_t^{NP}\right)\right\} = \mathbb{E}\left\{\sum_{t=0}^{\infty} \beta^t U\left(C_t^{\tau^s}, H_t^{\tau^s}\right)\right\}$, where the superscript "NP" indicates "no policy" and the superscript τ^s refers to the economy with the subsidy set at τ^s.

[45]For example, Benigno et al. (2013) and Gertler et al. (2012) set the risk aversion parameter to 2, as we do, and find the welfare gains from macroprudential policy to be well below 1 percent of consumption.

Table 6: Stochastic Steady State & Statistics with Macroprudential Subsidy

Variable	Symbol	Macroprudential Subsidy ($\tau^s = 0.56$)	No Policy
Output	Y	1.149	0.930
Consumption	C	0.859	0.720
Labor	L	0.410	0.339
Capital	K	9.322	7.192
Net worth	N	1.825	1.202
Equity issuance rate	x_t	0.034	0.023
Leverage ratio	$\frac{QS+(1-\Delta_L)S_W}{N}$	5.528	6.504
Maximum leverage ratio	ϕ	5.842	6.639
Spread (annualized, %)	$E(R_K) - R$	0.107	0.670
Foreign debt	B^*	1.119	0.901
Welfare gain (% of consumption)	Δ	1.27	–

Statistics

Prob. of bind. const., 2-qtr-ahead (%)		1.02	16.04
Prob. of bind. const., 1-yr-ahead (%)		4.30	35.69
Time at the constraint (% of qtrs)		2.06	13.76
Bind. const. event, mean duration (qtrs)		2.01	2.48
Crisis event, mean duration (qtrs)		6.00	5.82
Number of crisis events per 100 years		0.56	4.40

likelihood of the constraint binding in the future, even when in current conditions the constraint is slack, as in the steady state). The capital stock is thus considerably larger, leading to higher output and consumption. Finally, as shown by the last line in the Table, the subsidy does not completely eliminate the occurrence of financial crises, but their frequency is sharply reduced: the number of financial crisis events per 100 years goes down from more than four in the no-policy economy, to just above one-half in the economy with the subsidy.

6.2 Regulatory Capital Requirements

The second policy we consider is a regulatory capital requirement, in the spirit of actual Basel requirements. In particular, we consider a regulatory leverage limit, ϕ_t^R, on banks' risky-asset-to-net-worth ratio (which we call "risky leverage" to differentiate

it from total leverage $\frac{Q_t s_t + (1 - \Delta_L) s_{w,t}}{n_t}$). Accordingly, banks' problem now features an additional (regulatory) constraint (RC henceforth):

$$\frac{Q_t s_t}{n_t} \leq \phi_t^R \qquad \text{(RC)} \qquad (31)$$

We suppose that the regulatory agency imposes RC only in "normal times", i.e. only when the incentive compatibility constraint does not bind. We also assume that the regulatory limit ϕ_t^R depends on the aggregate state of the economy. In particular, ϕ_t^R is a function $\phi^R(\cdot)$ of the ratio $\frac{Q_t}{Q_t^*}$, where Q_t^* is the "unconstrained" asset price—the asset price that would realize in period t if the regulatory constraint was not binding (which would imply $\mu_t = 0$). This function is assumed to take the following form:

$$\phi_t^R = \phi^R \left(\frac{Q_t}{Q_t^*} \right) \equiv \begin{cases} \overline{\phi^R} & \text{if } \frac{Q_t}{Q_t^*} \geq 1 \\ \overline{\phi^R} \left(\frac{Q_t^*}{Q_t} \right)^\rho & \text{if } \frac{Q_t}{Q_t^*} < 1 \end{cases} \qquad (32)$$

with parameters $\overline{\phi^R}, \rho > 0$. Because a binding regulatory constraint will force the banking sector to hold less assets than otherwise, it will imply lower asset prices Q_t. Therefore, in equilibrium we will always have $Q_t \leq Q_t^*$, with equality when RC does not bind and with strict inequality when RC binds. Given these considerations, the function introduced in equation (32) has two features: first, the scalar $\overline{\phi^R}$ represents the limit for banks' risky leverage above which RC starts binding. Second, given a binding RC, the leverage limit ϕ_t^R adjusts upward as Q_t falls below Q_t^*, with an elasticity given by the parameter ρ.[46]

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[46]The latter feature is critical for an equilibrium to exist in periods when RC binds. The reason is the following: suppose that in period t, in the absence of RC banks would choose a risky leverage $\frac{Q_t K_t}{N_t}$ above $\overline{\phi^R}$. It follows that once RC is imposed, banks need to hold less assets than otherwise. In general equilibrium, this has the effect of pushing asset prices below what would otherwise obtain, as Q_t falls with aggregate investment. Lower asset prices, however, then imply lower net worth N_t. Further, given that banks were leveraged in the first place, the decline in N_t is typically so large that it more than offsets the decline in the value of assets $Q_t K_t$, with the result that total risky leverage actually *rises*—making leverage even higher, and therefore further away from (and not closer to) the regulatory leverage requirement. The presence of a large enough ρ in equation (32)—implying that the regulator "accommodates" declining asset prices a bit by relaxing RC—thus helps ensure that the equilibrium does not unravel due to the general-equilibrium drop in asset prices.

The above reasoning might suggest that asset prices could actually be higher, and not lower, with RC in order to restore equilibrium. This, however, is also not possible: note that our starting point was a state in which banks are unconstrained absent RC (implying $\mu_t = 0$). If asset prices were higher than that initial level, banks' excess returns μ_t would turn negative, and thus banks would rather not hold assets at all.

Figure 8: Effects of Regulatory Capital Requirement on Welfare ($\rho = 20$)
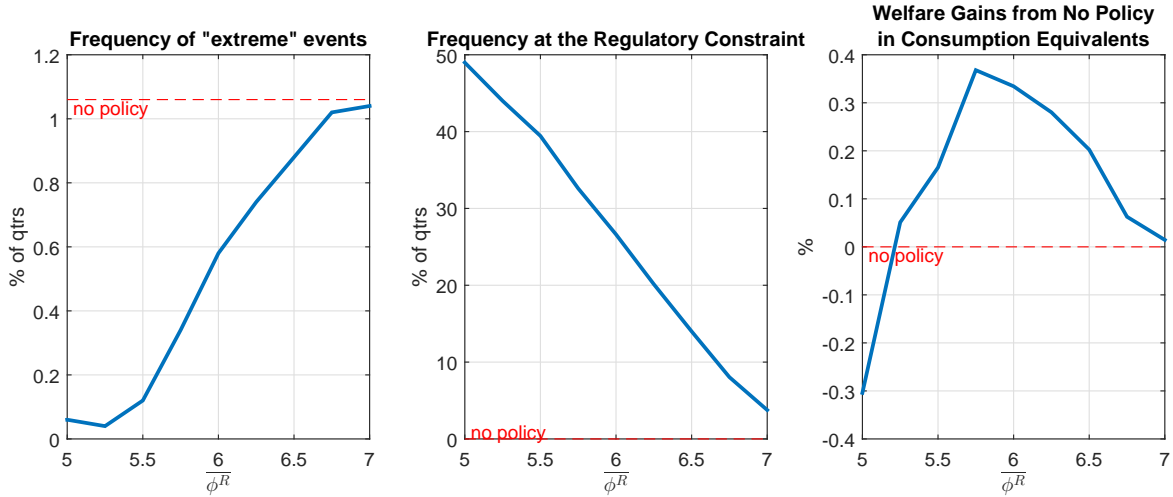
Note: The panels show, for a range of values of the regulatory leverage limit $\overline{\phi^R}$, the percentage of quarters when μ_t takes extreme values (left panel), the percentage of quarters spent at the regulatory constraint (middle panel) and the welfare gains from regulatory capital requirement in consumption equivalents (right panel).

The model simulation results with a regulatory capital requirement are presented in Figure 8. We solve the model globally with two occasionally binding constraints, the regulatory constraint on banks' risky leverage and the incentive compatibility constraint (stemming from the agency friction) on banks' total leverage. The left panel shows the frequency of events with "extreme" financial distress, defined as periods when banks' excess return on risky assets, μ_t, is above a threshold value of 0.049,[47] as well as the frequency at the regulatory constraint (middle panel) for different values of the regulatory parameter $\overline{\phi^R}$. The same charts also display these magnitudes in the no-policy economy (dashed red line) for a comparison. We pick values of $\overline{\phi^R}$ changing from 5.25 to 7, with the average value of the risky leverage ratio in the no-policy economy, at 6, falling around the middle of this range. We set ρ at a baseline value of 20, which ensures existence of equilibrium throughout the simulation, but emphasize that results are robust to variations around that value (so long as ρ is large enough to avoid existence problems).

The economy typically spends a considerable amount of time at the regulatory

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[47]We pick the threshold value as one standard deviation above the average value of μ_t in the no-policy economy, conditional on $\mu_t > 0$.

capital constraint. For example, when $\overline{\phi^R} = 5.75$, banks are up against the regulatory constraint 33 percent of quarters. This, in turn, ensures that banks are less likely to face severe financial crisis events when the regulatory limit is imposed than when it is not (0.3 percent when $\overline{\phi^R} = 5.75$, versus 1.06 percent without policy). The intuition for this result is simple: the presence of the regulatory leverage constraint works to lower banks' risky leverage on average, and at the same time creates more frequent instances in which excess returns take small positive values. Both effects tend to make events with extremely low net worth, in which incentive constraints bind and excess returns shoot up, less likely. Thus, the higher the frequency spent at the regulatory constraint (as one moves from right to left in the middle panel), the lower the chance of facing events with severe financial stress (moving toward the left in the first panel).

Next, we calculate welfare gains from regulatory capital requirement policy. As in the case of government subsidy, our metric is the unconditional expectation of welfare: for each value of the policy $\overline{\phi^R}$, we compute consumption-equivalent welfare gains by finding the permanent increase in quarterly consumption in the no-policy economy required to make the household as well off as in the economy with policy. We find that the regulatory policy raises welfare. The welfare gains are small but positive for the most loose regulatory limit that we consider, $\overline{\phi^R} = 7$, and then rise gradually as we make $\overline{\phi^R}$ tighter. Along the way, the frequency of banks being constrained by RC rises, and the frequency of extreme financial stress falls. Our simulations suggest that the welfare is maximized when the regulatory leverage limit is set at 5.75, with the welfare gains diminishing as the limit becomes tighter.

Note that for values of $\overline{\phi^R}$ below 5.75, the welfare gain starts decreasing, and even turn negative when the regulatory limit is very tight. Because banks' ability to finance risky capital investment is hindered by the regulatory constraint, the economy might end up with very low levels of physical capital when the capital requirement is very tight, which might be detrimental for welfare (even if it does reduce the frequency of occurrence of financial crises). Hence, in determining the intensity of capital requirements the regulatory agency has to evaluate the tradeoff between disrupting the accumulation of physical capital stock and the probability of a financial crisis.

7   Conclusion

We have developed a small open economy framework with banks that face occasionally binding leverage constraints. The latter feature implies that the model can generate the type of nonlinear dynamics usually associated with banking crises. The model produces episodes of financial crisis nested within normal business cycle fluctuations, and does not need to rely on unusually large shocks to produce a crisis. A virtue of our approach is that by analyzing a fully nonlinear solution, we can adequately capture the risk-taking behavior of banks. Moreover, by allowing banks to issue equity, we can capture how banks endogenously adjust the strength of their balance sheet in response to economic conditions.

We can also examine whether there is a role for macroprudential policy in enhancing financial stability. We show that an appropriate government subsidy policy is effective in reducing the probability of financial crises, thereby improving upon *laissez faire*. We also show bank capital requirements might be welfare improving. In particular, our results suggest that in determining the intensity of capital requirements the regulatory agency has to evaluate the tradeoff between disrupting the accumulation of physical capital stock and the probability of a financial crisis.

Our focus has been to produce a framework that is tractable enough to accommodate easily the features used in the DSGE literature to enhance the quantitative performance and to facilitate policy analysis. Capital requirements are not the only tool through which the policymaker can reduce the inefficiencies arising from pecuniary externalities. Our model can also provide useful insight into the relative benefits of capital controls *vis-á-vis* bank capital requirements. Finally, another interesting avenue of future research would be to augment the model with nominal rigidities, and use it to analyze the implications of financial stability considerations for the conduct of monetary policy. We are working on those extensions.

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Appendix (For Online Publication)

A Crisis Event Study for the Extended Sample

Figure 9: Crises Event Study
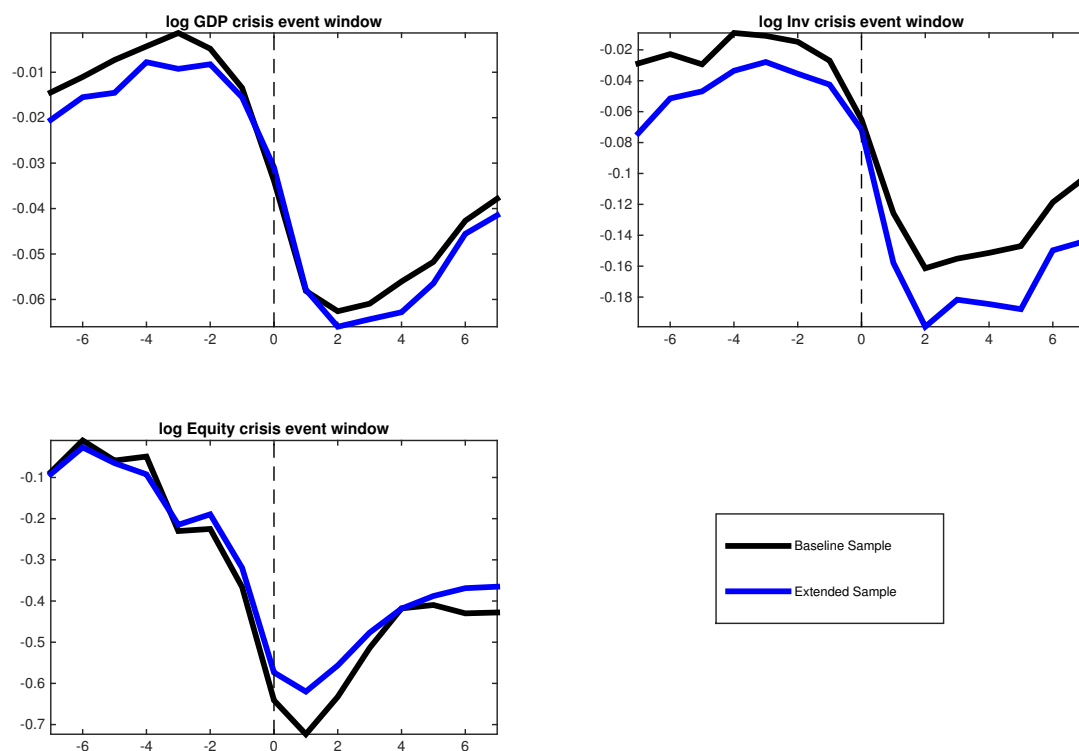
Note: Bank equity, GDP and investment are all deflated by GDP deflator, and HP detrended with a smoothing parameters 1600. The events are centered at the systemic banking crises episodes (time t=0) identified by Laeven and Valencia (2012) for each country. There are twenty-three systemic banking crises episodes in the extended sample, compared to eight in the baseline sample. The baseline sample includes data for the following (crisis start: year in parenthesis): Italy (2008), Spain (2008), Germany (2008), France (2008), UK (2007), and the US (2008). The extended sample includes Austria (2008), Belgium (2008), Denmark (2008), France (2008), Germany (2008), Greece (2008), Iceland (2008), Ireland (2008), Italy (2008), Japan (1997), Luxembourg (2008), Netherlands (2008), Norway (1991), Portugal (2008), Spain (2008), Sweden (1991), Switzerland (2008), UK (2007), US (1988, 2007), Korea (1997), Mexico (1994), and Turkey (2000). The event window includes 7 quarters before and 7 quarters after the event, and all series are normalized at their respective pre-crisis peaks.

B Household's Optimality Conditions

$$U_1(C_t, H_t) = U_{C,t} \tag{33}$$

$$-U_2(C_t, H_t) = U_{C,t} W_t \tag{34}$$

$$\mathbb{E}_t(\Lambda_{t,t+1} R_t) = 1 \tag{35}$$

The household's stochastic discount factor is defined as

$$\Lambda_{t,t+1} = \beta \frac{U_{C,t+1}}{U_{C,t}} \tag{36}$$

where $U_{C,t}$ is the marginal utility of consumption.

C Details on Working Capital Loans

We follow the timing assumptions in Neumeyer and Perri (2005). Within each period t there are two subperiods: t^- and t^+. Period-t shocks are revealed in t^-. Times t^+ and $(t+1)^-$ are arbitrarily close. As in Neumeyer and Perri (2005), final goods producers hire labor and capital at time t^-, and output becomes available in t^+. Firms need to set aside fraction Υ of the wage bill in t^- (before production takes place), and so they need to borrow this amount. In contrast to Neumeyer and Perri (2005), we assume firms cannot borrow directly from international financial markets (or from domestic households), but need to borrow from domestic financial intermediaries. The interest charged by banks on working capital loans taken out in t^- is denoted $R_{L,t}$. These loans are repaid at time t^+, after final goods become available.

In period t^-, banks borrow $d_{W,t}$ in the international noncontingent debt market, at interest rate R_{t-1}. Noncontingent bonds issued at time t^- or at time $(t-1)^+$ mature at time t^+. Risky loans for investment financing, s_{t-1}, also mature at t^+, once nonfinancial firms' output becomes available. Equity issued in the previous period also becomes available at time t^+. Accordingly, banks do not have internal funds at time t^-, and so working capital loans—denoted $s_{W,t}$—are financed entirely by noncontingent debt: $s_{W,t} = d_{W,t}$. The net proceeds to the bank from working capital lending are therefore given by $(R_{L,t} - R_{t-1}) s_{W,t}$. These proceeds become available to

the bank at time t^+. All told, the bank's period-t budget constraint (assuming the bank does not default) is given by equation (6) in the main text.

We assume that if banks default, they do so on both deposits issued at t^- (for working capital loans) and at t^+ (to finance investment lending). At the end of the period, the defaulting bank is forced into bankruptcy. The recovery rates for working capital assets and for physical capital assets are allowed to differ in principle (although as a plausible benchmark, in our calibration we set them to be equal). Accordingly, the bank can divert fraction ω of resources to finance working capital loans, and fraction θ of resources to finance risky lending. The total payoff for a defaulting bank is thus given by the following expression:

$$\theta \left[Q_t s_t - (R_{L,t} - R_{t-1}) s_{W,t} \right] + \omega s_{W,t} \tag{37}$$

Here the term in square brackets multiplying θ represents the amount the bank can divert at t^+, given that it has defaulted on working capital loans, thereby forgoing the net proceeds from these loans. From the balance sheet equation, this term equals to $d_t + n_t$; i.e., time-t^+ borrowing plus net worth. The expression above then gives rise to the incentive constraint as given by equation (3) in the main text.

D Banker's Problem

Under our functional form assumption for $C(e_t, n_t)$, the banker's problem is:

$$\alpha_t n_t = \max_{s_t, s_{W,t}, x_t} \mu_t Q_t s_t + \nu_t \Delta_{L,t} s_{W,t} + \nu_t n_t + \sigma n_t (\nu_{e,t} x_t - \frac{\kappa}{2} x_t^2) \tag{38}$$

subject to

$$\mu_t Q_t s_t + \nu_t \Delta_{L,t} s_{W,t} + \nu_t n_t + \sigma n_t (\nu_{e,t} x_t - \frac{\kappa}{2} x_t^2) \geq \theta \left[Q_t s_t - \Delta_{L,t} s_{W,t} \right] + \omega s_{W,t} \tag{39}$$

The first order condition for x_t is $\nu_{e,t} = \kappa x_t$. Imposing this condition, the bank's problem becomes:

$$\alpha_t n_t = \max_{s_t, s_{W,t}} \mu_t Q_t s_t + \nu_t \Delta_{L,t} s_{W,t} + (\nu_t + \frac{\sigma \kappa}{2} x_t^2) n_t \tag{40}$$

subject to

$$\mu_t Q_t s_t + \nu_t \Delta_{L,t} s_{W,t} + (\nu_t + \frac{\sigma\kappa}{2}x_t^2)n_t \geq \theta\left[Q_t s_t - \Delta_{L,t} s_{W,t}\right] + \omega s_{W,t} \tag{41}$$

Then the Lagrangian can be written as:

$$\mathcal{L}_t = (1 + \lambda_t)\left[\mu_t Q_t s_t + \nu_t \Delta_{L,t} s_{W,t} + (\nu_t + \frac{\sigma\kappa}{2}x_t^2)n_t\right] - \lambda_t\left[\theta Q_t s_t + (\omega - \theta\Delta_{L,t})s_{W,t}\right] \tag{42}$$

where λ_t is the multiplier on the incentive constraint.

E   Resource Constraint and Balance of Payments

Aggregating the bank's budget constraint across banks and combining it with the household's budget constraint along with the market clearing condition for claims on capital $(S_t = K_t)$, we obtain:

$$Q_t K_t + R_{t-1}B_{t-1}^* + C_t + \sigma\frac{\kappa}{2}x_t^2 N_t \leq W_t H_t + R_{K,t}Q_{t-1}K_{t-1}$$
$$+ (R_{L,t} - R_{t-1})S_{W,t} + Q_{t-1}K_{t-1} + B_t^* + \Pi_t^F + \Pi_t^C \tag{43}$$

The last two terms, Π_t^F and Π_t^C, are the profits of final goods firms and capital producers, respectively. They are given by their respective budget constraints:

$$Y_t + Q_t\left(1 - \delta\right)e^{\psi_t}K_{t-1} = \Pi_t^F + W_t H_t(1 + \Upsilon(R_{L,t} - 1)) + R_{K,t}Q_{t-1}K_{t-1} \tag{44}$$

$$\Pi_t^C = Q_t I_t - \left[1 + f\left(\frac{I_t}{I_{t-1}}\right)\right]I_t \tag{45}$$

Using these expressions, we can derive the resource constraint and the balance of payments equation for the economy as the following:

$$Y_t = C_t + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right)\right]I_t + \sigma\frac{\kappa}{2}x_t^2 N_t + NX_t \tag{46}$$

$$R_{t-1}B_{t-1}^* - B_t^* + (R_{t-1} - 1)\Upsilon W_t H_t = NX_t \tag{47}$$

F   Full Set of Equilibrium Conditions

$$Y_t = C_t + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right)\right] I_t + \sigma\frac{\kappa}{2}x_t^2 N_t + NX_t \tag{48}$$

$$NX_t = R_{t-1}B_{t-1}^* - B_t^* + (R_{t-1} - 1)\Upsilon W_t H_t \tag{49}$$

$$K_t = I_t + (1 - \delta)e^{\psi_t}K_{t-1} \tag{50}$$

$$Q_t = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}}f'\left(\frac{I_t}{I_{t-1}}\right) - E_t\Lambda_{t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 f'\left(\frac{I_{t+1}}{I_t}\right) \tag{51}$$

$$1 = \mathbb{E}_t\left(\Lambda_{t+1}\right)R_t \tag{52}$$

$$\Lambda_t = \beta\left(\frac{C_t - \chi\frac{H_t^{1+\epsilon}}{1+\epsilon}}{C_{t-1} - \chi\frac{H_{t-1}^{1+\epsilon}}{1+\epsilon}}\right)^{-\gamma} \tag{53}$$

$$R_{K,t} = e^{\psi_t}\frac{\alpha\frac{Y_t}{e^{\psi_t}K_{t-1}} + (1-\delta)Q_t}{Q_{t-1}} \tag{54}$$

$$Y_t = A_t(e^{\psi_t}K_{t-1})^\eta H_t^{1-\eta} \tag{55}$$

$$\mu_t = \mathbb{E}_t\left[\Lambda_{t+1}(1 - \sigma + \sigma\alpha_{t+1})\left(R_{K,t+1} - R_t\right)\right] \tag{56}$$

$$\nu_t = \mathbb{E}_t\left[\Lambda_{t+1}(1 - \sigma + \sigma\alpha_{t+1})\right]R_t \tag{57}$$

$$\nu_{e,t} = \mathbb{E}_t\left[\Lambda_{t+1}\left(\alpha_{t+1} - 1\right)\right] \tag{58}$$

$$\alpha_t = \mu_t\phi_t + \nu_t + \sigma\frac{\kappa}{2}x_t^2 \tag{59}$$

$$N_t = \sigma[(R_{K,t} - R_{t-1})Q_{t-1}K_{t-1} + x_{t-1}N_{t-1} \tag{60}$$
$$+ R_{t-1}\left(N_{t-1} + \Delta_{L,t-1}\Upsilon W_{t-1}H_{t-1}\right)] + (1-\sigma)\xi Q_{t-1}K_{t-1}$$

$$W_t = \chi H_t^\epsilon \tag{61}$$

$$\frac{Y_t}{H_t} = \frac{W_t\left[1 + \Upsilon\left(R_{L,t} - 1\right)\right]}{(1-\eta)} \tag{62}$$

$$R_t = \frac{1}{\beta} + \varphi\left(e^{\frac{B_t^*}{Y_t} - \bar{b}} - 1\right) + e^{R_t^* - 1} - 1 \tag{63}$$

$$\phi_t = \frac{\nu_t + \sigma\frac{\kappa}{2}x_t^2}{\theta - \mu_t} \tag{64}$$

$$x_t = \frac{\nu_{e,t}}{\kappa} \tag{65}$$

$$R_{L,t} = R_{t-1} + \Delta_{L,t} \tag{66}$$

where $\Delta_{L,t} = \frac{\mu_t}{\mu_t + \nu_t}$. If the constraint binds, we must have

$$Q_t S_t + (1 - \Delta_{L,t})\Upsilon W_t H_t = \phi_t N_t \qquad (\text{and } \mu_t > 0) \qquad (67)$$

If the constraint does not bind, we have

$$\mu_t = 0 \qquad (\text{and } Q_t S_t + (1 - \Delta_{L,t})\Upsilon W_t H_t < \phi_t N_t) \qquad (68)$$

G Solution Method

Let $\overline{K}_t \equiv e^{\psi_t} K_{t-1}$ denote the effective amount of physical capital at the beginning of period t (after the capital quality shock is realized), and define $\overline{B}_{t-1} \equiv R_{t-1} B^*_{t-1}$ to be the stock of external debt plus interest. Let also $\overline{N}_{t-1}$ refer to the predetermined part of aggregate net worth (i.e., the component of net worth that does not depend on time-t variables like Q_t), given by the following:

$$\overline{N}_{t-1} = \sigma \left[x_{t-1} N_{t-1} + R_{t-1} \left(\underbrace{N_{t-1} + \Delta_{L,t}\Upsilon W_{t-1} H_{t-1} - Q_{t-1} K_{t-1}}_{=-D_{t-1}} \right) \right] + (1-\sigma)\xi Q_{t-1} K_{t-1}$$

Note that $\overline{N}_{t-1}$ is equal to aggregate new equity issued by surviving banks $(\sigma x_{t-1} N_{t-1})$, plus startup transfers to entering banks $((1 - \sigma)\xi Q_{t-1} K_{t-1})$, minus the total stock of debt (with interest) carried over by surviving banks $(\sigma R_{t-1} D_{t-1})$. Given our calibration the latter term will always be large relative to the first two, so that $\overline{N}_{t-1} < 0$.

Given these definitions, let $\mathbf{S}_t$ denote the model's aggregate state vector, given by seven variables:

$$\mathcal{S}_t \equiv \left\{ \overline{K}_t, -\overline{N}_{t-1}, \overline{B}_{t-1}, R_{t-1}, I_{t-1}, R^*_t, A_t \right\}$$

We use the negative of $\overline{N}_{t-1}$ so that $\mathcal{S}_t > 0$. Following the parameterized expectations approach (PEA henceforth), our solution method relies on using parametric functions to approximate the model's one-step-ahead expectations. To this end, define the following objects:

$$\mathbb{E}_{1,t} \equiv \mathbb{E}_t \left(U_{C,t+1} \right) R_t \tag{69}$$

$$\mathbb{E}_{2,t} \equiv \mathbb{E}_t \left[U_{C,t+1} \left(1 - \sigma + \sigma \alpha_{t+1} \right) \left(\frac{\alpha Y_{t+1}}{e^{\psi_{t+1}} K_t} + (1 - \delta) Q_{t+1} \right) \right] \tag{70}$$

$$\mathbb{E}_{3,t} \equiv \mathbb{E}_t \left[U_{C,t+1} \left(1 - \sigma + \sigma \alpha_{t+1} \right) \right] R_t \tag{71}$$

$$\mathbb{E}_{4,t} \equiv \mathbb{E}_t \left[U_{C,t+1} \left(\nu_{t+1} + \mu_{t+1} \phi_{t+1} + \frac{\sigma \kappa}{2} x_{t+1}^2 \right) \right] \tag{72}$$

$$\mathbb{E}_{5,t} \equiv \mathbb{E}_t \left(U_{C,t+1} \right) \tag{73}$$

$$\mathbb{E}_{6,t} \equiv \mathbb{E}_t \left[U_{C,t+1} \left(\frac{I_{t+1}}{I_t} \right)^3 \right] \tag{74}$$

$$\mathbb{E}_{7,t} \equiv \mathbb{E}_t \left[U_{C,t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 \right] \tag{75}$$

Our choice of expectations reflects two main considerations: First, it should be possible to approximate them accurately with a parameterized function of the state. Second, they should facilitate the solving of the model. We have found that the expectations above satisfy both criteria successfully: as we illustrate below, they are in general smooth functions of the state; in addition, it is straightforward to show that knowing $\mathcal{S}_t$ and $\{E_{i,t}\}_{i=1}^{7}$ one can retrieve all time-t endogenous variables (as well as the evolution of the state vector) in closed form from (48)-(68) when the constraint does not bind; in addition, one can collapse the system (48)-(68) to just one nonlinear equation in one unknown (Q_t or μ_t) when the constraint binds.

The problem is then to find the set of functions of the aggregate state $\varepsilon_i(\mathcal{S}_t) : \mathbb{R}^7 \to \mathbb{R}$ for $i = 1, ...7$ that determine the expectations (69)-(75). To approximate the ε_i functions, we use third-order polynomials for the log of the conditional expectations in the log of the state, following Den Haan (2007). That is, letting $P_3(s; \varrho)$ stand for the third-order polynomial in the vector s with coefficients ϱ, we approximate the functions as $\varepsilon_i(\mathcal{S}_t) \approx e^{P_3(\log(\mathcal{S}_t); \varrho_i)}$. We have found that the exponential-log formulation enhances accuracy significantly compared to a standard polynomial. All told, we need to find the 120 coefficients in ϱ_i for each $i = 1, ..., 7$. Our algorithm adapts PEA by using quadrature to compute expectations, as advised by Judd, Maliar and Maliar (2011) and Den Haan (2007). As these authors point out, this approach significantly enhances accuracy and speed. We continue to use simulation-based PEA: also as

pointed out by Judd, Maliar and Maliar (2011) and Den Haan (2007), a virtue of this approach (as opposed to solving the model on a pre-specified grid) is that one needs to solve the model only in points in the state space that are actually visited in equilibrium.

Let $\boldsymbol{\varrho} \equiv (\varrho_i)_{i=1}^7$. Our algorithm proceeds as follows:

0. Let $\boldsymbol{\varrho}^0$ be the initial set of coefficients in this step. Simulate the model for 5,000 periods by solving the system (48)-(68) characterizing equilibrium, given a sequence of realizations of the exogenous innovations (and setting $\mathcal{S}_0$ at the steady-state value of the state vector). To do so, at each period t we first solve the system assuming that the constraint does not bind (implying $\mu_t = 0$). We then check if bank leverage is above the maximum allowed by the constraint. If it is not, we proceed; if it is, we again solve the system, this time imposing that the constraint binds.

   • At each t, after solving for the equilibrium, we approximate the set of one-step-ahead conditional expectations (69)-(75) by Gauss-Hermite quadrature.

1. Obtain a new set of coefficients $\boldsymbol{\varrho}^1$ by regressing the (log of the) conditional expectations obtained in the previous step on $P_3 (\log(\mathcal{S}_t); \cdot)$, where $\mathcal{S}_t$ is the state vector from the simulation in the previous step.

2. Compare $\boldsymbol{\varrho}^1$ with $\boldsymbol{\varrho}^0$. If they are close enough, stop. If not, update $\boldsymbol{\varrho}^0$ by setting $\boldsymbol{\varrho}^0 = \lambda_\varrho \boldsymbol{\varrho}^1 + (1 - \lambda_\varrho) \boldsymbol{\varrho}^0$ and go back to step 0. We have found a value of 2/3 for the "smoothing" parameter λ_ϱ to work well in our setting.

To initialize our algorithm we need an initial value for $\boldsymbol{\varrho}^0$. To this purpose, we first simulate the model using the OccBin toolkit developed by Guerrieri and Iacoviello (2015), and regress the resulting conditional expectations on the state vector to initialize $\boldsymbol{\varrho}^0$. This approach has proved very helpful to obtain fast convergence of our algorithm.

# H Euler Residuals and Expectation Functions Fit

Following Judd (1992), we provide a check on the accuracy of our solution method by computing Euler equation errors. Moving from the Euler equation for consumption, we define the Euler equation error (as a fraction of units of consumption) as

$$err_t = \left| \frac{[\beta \mathbb{E}_t(U_{C,t+1})R_t]^{-\frac{1}{\gamma}} + \frac{\chi}{1+\epsilon}H_t^{1+\epsilon} - C_t}{C_t} \right| \qquad (76)$$

Above, we again approximate $\mathbb{E}_t(U_{C,t+1})$ by Gauss-Hermite quadrature. Figure 10 shows a histogram of the errors for a given simulation. We express the errors in decimal log scale, as is common in the literature. The Euler errors are reasonably small, and comparable to those found in the literature. The average error is about -5.15. To interpret, recall that under the decimal log scale a value of, say, -5 is that the error is sized at \$1 per \$100,000 of consumption.

Figure 10: Histogram of Euler Residuals
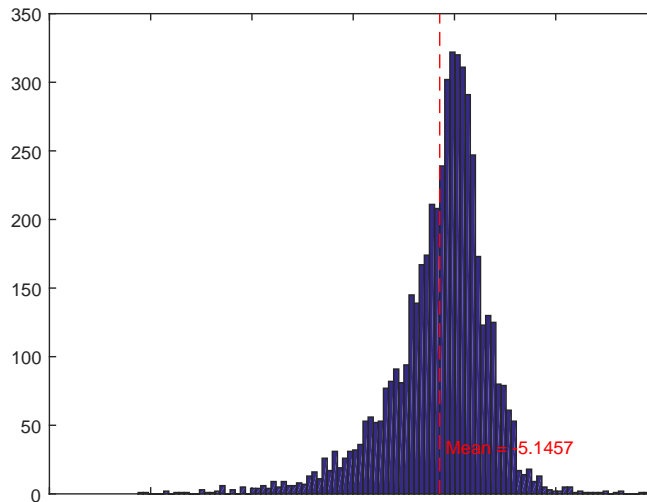
Note: Histogram of Euler equation errors in model simulation.

Table 7: R^2 of Expectation Regressions

$\mathbb{E}_{1,t}$	$\mathbb{E}_{2,t}$	$\mathbb{E}_{3,t}$	$\mathbb{E}_{4,t}$	$\mathbb{E}_{5,t}$	$\mathbb{E}_{6,t}$	$\mathbb{E}_{7,t}$
1.0000	0.9982	0.9979	0.9979	1.0000	0.9995	0.9998

As a second accuracy check, we verify that the expectations are approximated accurately by examining goodness-of-fit measures of the regression in step 1 of our algorithm (see the previous section). Table 7 provides the values of R^2 for each of the expectations (69)-(75). The R^2's are either unity or very close to unity (the smallest one equals 0.9979), indicating a good fit of the parameterized expectation functions.

I Model Policy and Expectation Functions

Figures 11 through 14 show different slices of the model's policy functions for a set of the model's endogenous variables. Note that moving toward adverse values of the state (for example, toward lower values of $\overline{K}_t$ in Figure 11, or lower values of $\overline{N}_{t-1}$ in Figure 12, or higher values of $\overline{B}_{t-1}$ in Figure 13) net worth declines and eventually the constraint starts binding: μ_t turns positive, and the declines in net worth, Tobin's Q, investment and output turn steeper. Note also that as $\overline{K}_t$ declines (or $\overline{N}_{t-1}$ declines, or $\overline{B}_{t-1}$ increases), leverage, ϕ_t, increases, and so does equity issuance, x_t. Similar observations apply when one varies the remaining states (not shown). In addition, the constrained region is not only characterized by very low values of $\overline{K}_t$ or $\overline{N}_{t-1}$, but also by a combination of relatively low values of both, as shown by Figure 14. Overall, note that the model's policy functions tend to be highly nonlinear, displaying sharp kinks when the constraint starts binding.

By contrast, the model's expectation functions tend to be much smoother. As an example, Figures 15 and 16 show the model's one-step-ahead conditional expectations (69)-(75) as functions of the state variables $\overline{K}_t$ and $\overline{N}_{t-1}$. Note that the conditional expectations (shown by the blue solid line) in general do not display the kinks exhibited by the policy functions. Note also that our polynomial approximation (the orange dashed line) tracks very well the actual expectations, in line with the very high R^2's reported in the previous section.

J Computation of Binding Constraint Probabilities

At period t_0, we compute the probability of the constraint binding at least once within horizon $t_0 + j$ as follows. First, obtain draws for the exogenous innovations $\{\psi_t, \epsilon_{R,t}, \epsilon_{A,t}\}_{t_0+1}^{t_0+j}$, together with their associated probabilities. For each history of realizations of shocks h, defined as each possible sequence of realizations of $\{\psi_t, \epsilon_{R,t}, \epsilon_{A,t}\}_{t_0+1}^{t_0+j}$, let the set of histories in which there is at least one crisis be H. Then the probability of a crisis (for a given horizon j) is the sum of the probabilities of each of the histories in H, i.e. $\sum_{h \in H} p(h)$. In the body of the paper we report the results for $j = 2, 4$.

Figure 11: Model Policy Functions (I): $\overline{K}_t = e^{\psi_t} K_{t-1}$
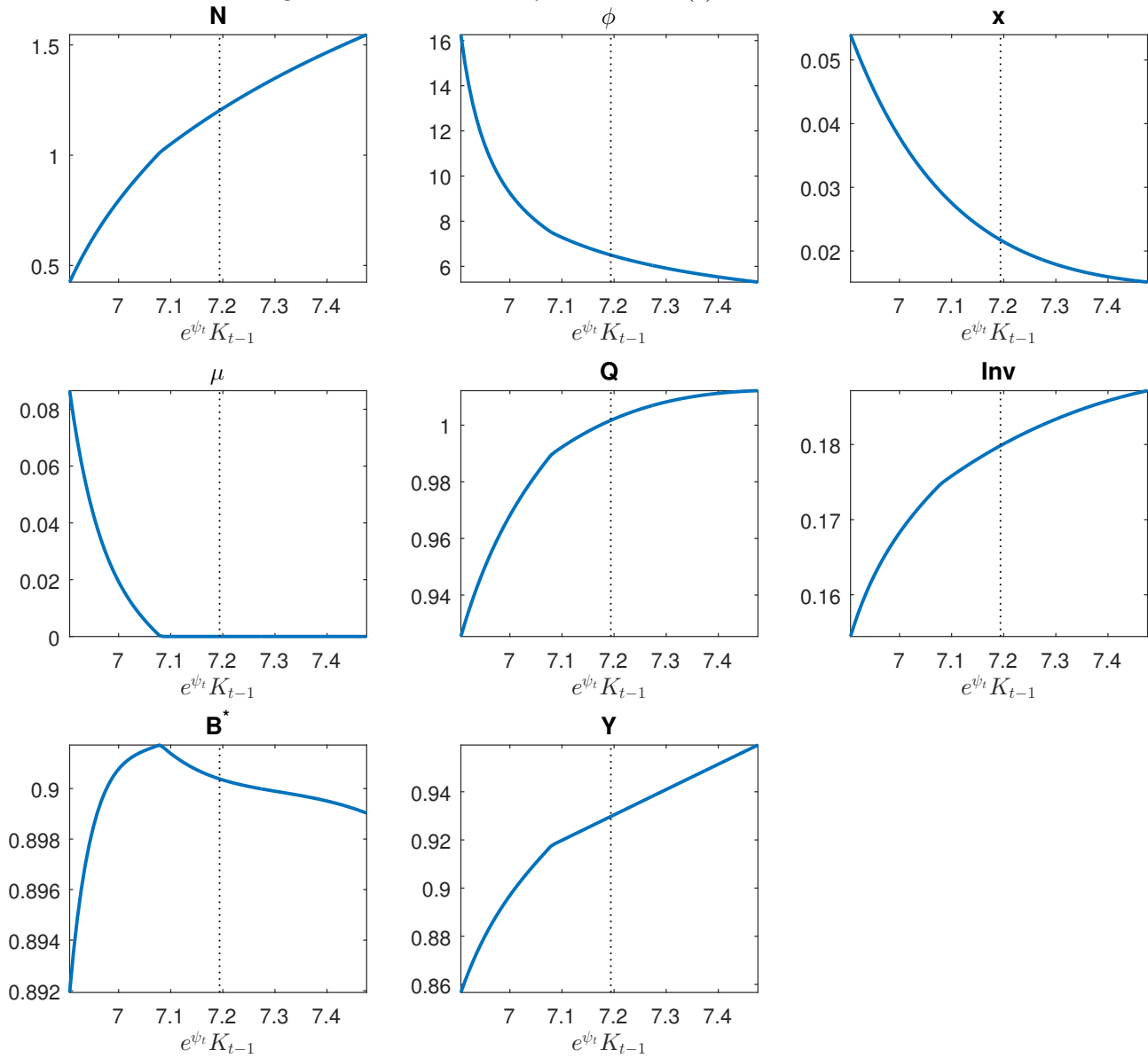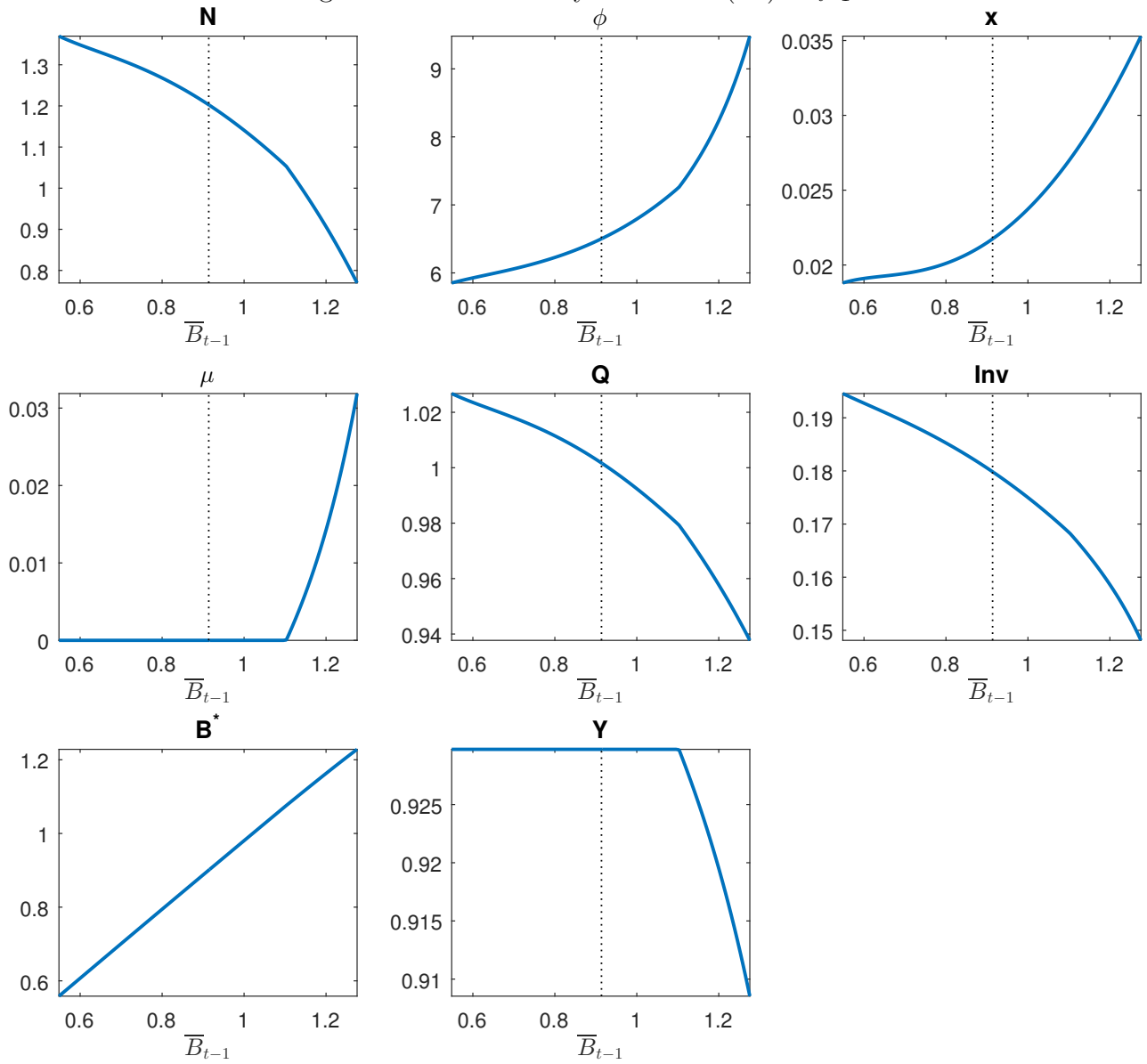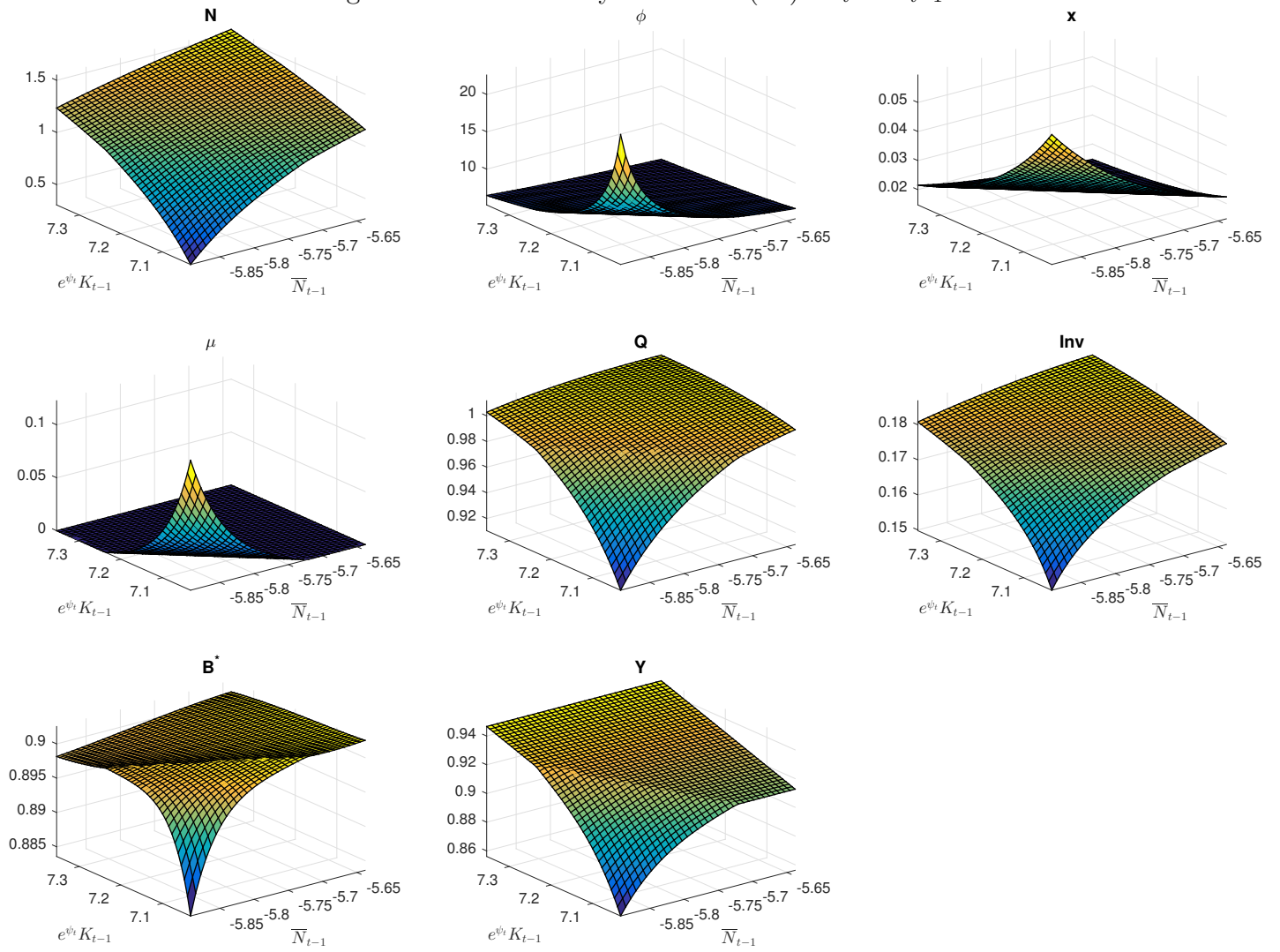
Note: Model endogenous variables as a function of state variable $\overline{K}_t$. All other states kept at stochastic-steady-state value. Dotted vertical line indicates stochastic-steady-state value of $\overline{K}_t$.

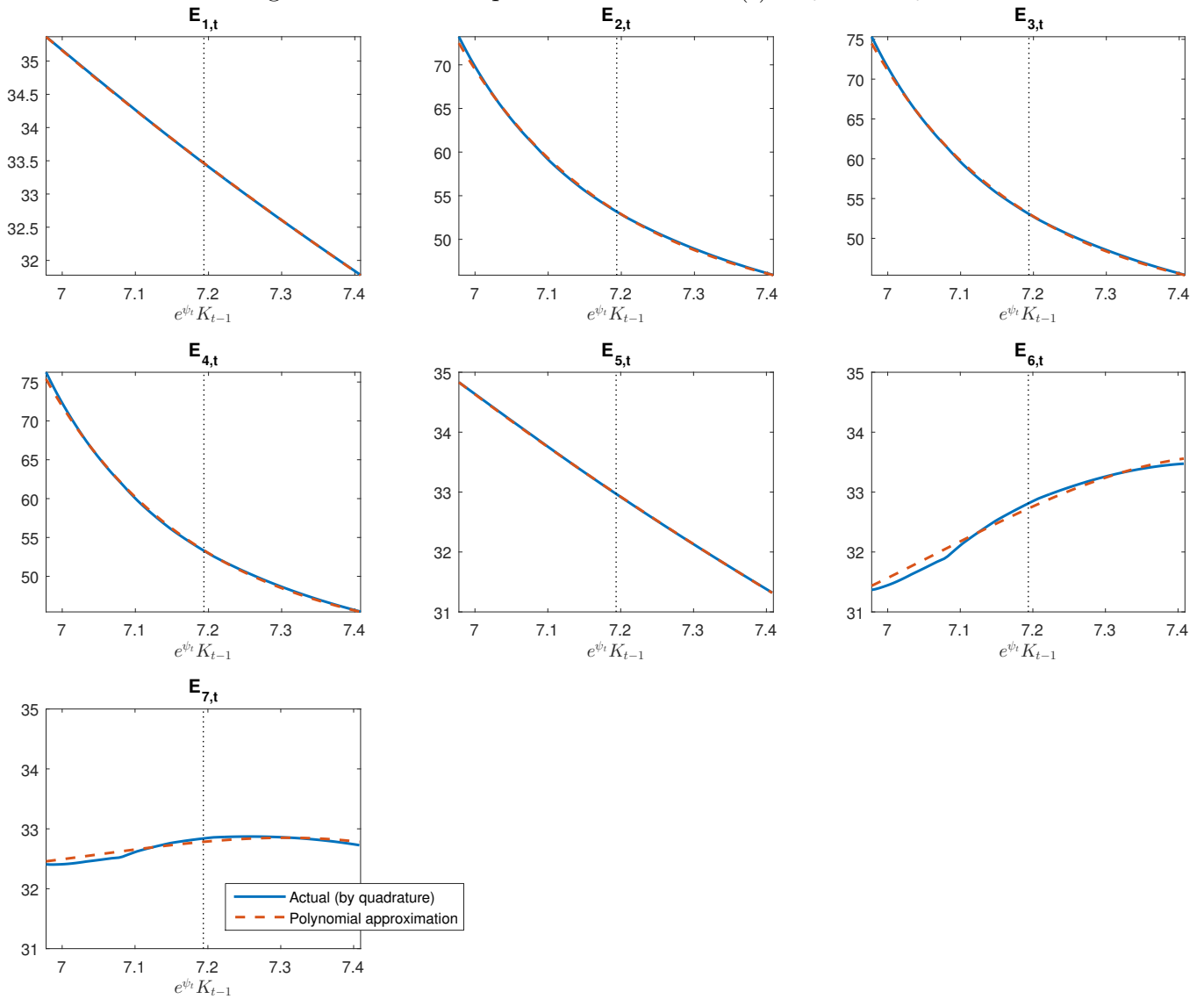Figure 12: Model Policy Functions (II): $\overline{N}_{t-1}$
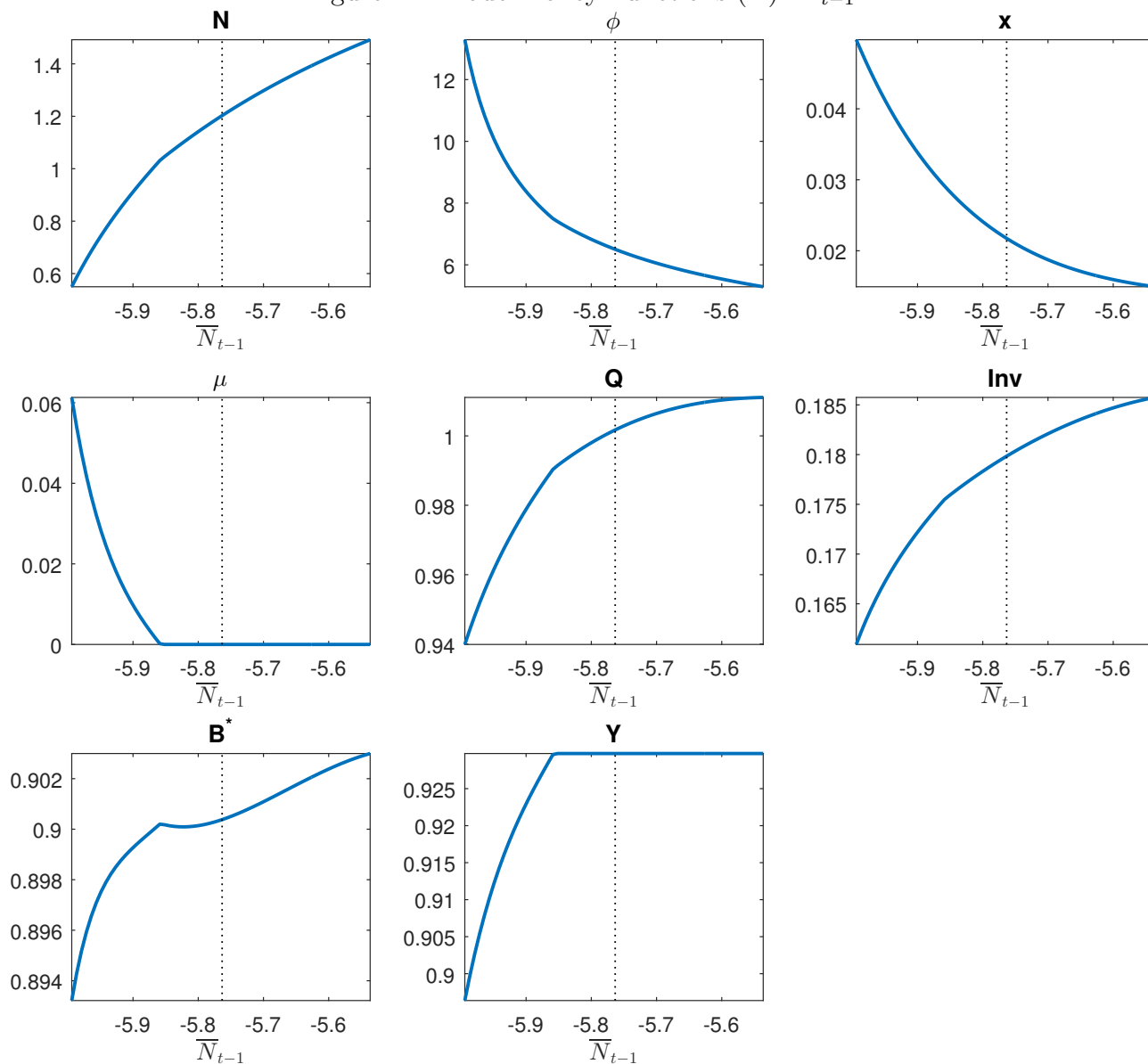
Note: Model endogenous variables as a function of state variable $\overline{N}_{t-1}$. All other states kept at stochastic-steady-state value. Dotted vertical line indicates stochastic-steady-state value of $\overline{N}_{t-1}$.

Figure 13: Model Policy Functions (III): $\overline{B}_{t-1}$

Note: Model endogenous variables as a function of state variable $\overline{B}_{t-1}$. All other states kept at stochastic-steady-state value. Dotted vertical line indicates stochastic-steady-state value of $\overline{B}_{t-1}$.

Figure 14: Model Policy Functions (IV): $\overline{K}_t \times \overline{N}_{t-1}$
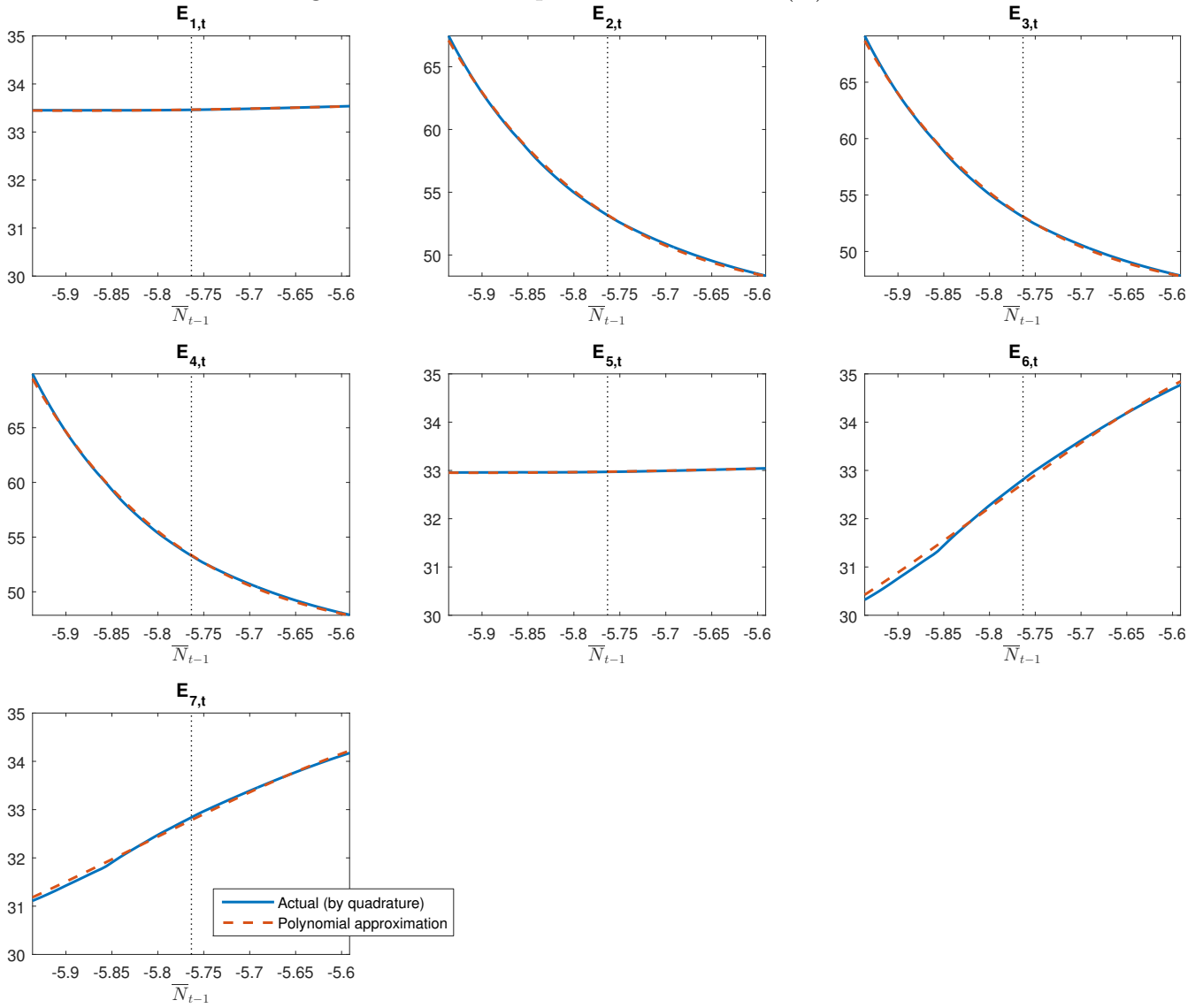
Note: Model endogenous variables as a function of $\overline{K}_t$ and $\overline{N}_{t-1}$. All other states kept at stochastic-steady-state value.

Figure 15: Model Expectation Functions (I): $\overline{K}_t = e^{\psi_t} K_{t-1}$

Note: Model expectations as a function of state variable $\overline{K}_t$. All other states kept at stochastic-steady-state value. Dotted vertical line indicates stochastic-steady-state value of $\overline{K}_t$.

Figure 16: Model Expectation Functions (II): $\overline{N}_{t-1}$

Note: Model expectations as a function of state variable $\overline{N}_{t-1}$. All other states kept at stochastic-steady-state value. Dotted vertical line indicates stochastic-steady-state value of $\overline{N}_{t-1}$.